# Spinor matter waves: Modulational instability vs. nonlinearity

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In the recent years there has been a growing interest in the dynamics of spin-1 Bose-Einstein condensate composed of atoms in three different hyperfine states $m_F = 0, \pm 1$ and trapped in an optical dipole trap or an optical lattice. The rich dynamics of the spinor matter waves, which includes domain separation, formation of spatial patterns and three-component localized states is partly due to the dynamical (modulational) instabilities that occur in this nonlinear system with complex intra- and inter-component interactions. It is therefore of critical importance to understand details of the interplay between the instabilities and the effect of nonlinearity.

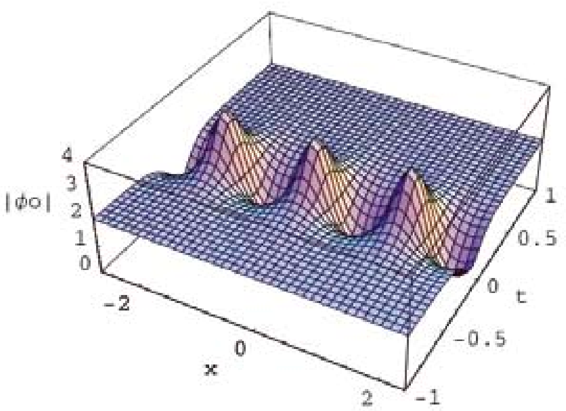


Figure 1: Time evolution of the density of the $m_F = 0$ spinor-component in a lattice-free, quasi-1D sigar-shaped trap. The densities of the $m_F = \pm 1$ components oscillate in phase with the $m_F = 0$ component.

To investigate the details of the spinor dynamics beyond the onset of modulational instability, we have considered a mean-field model of a ferromagnetic spinor condensate with attractive interactions confined in a quasi-1D sigar-shaped trap. Under certain assumptions, this model can be reduced to a fully integrable matrix Nonlinear Schrödinger equation [1], which lends itself to a fully analytical analysis. Our study revealed that the exponential growth of small-amplitude perturbations due to the modulational instability of a spinor condensate is halted by the emergence of the nonlinearly localized structures and replaced by an exponential decay. The overall evolution therefore demonstrates the Fermi-Pasta-Ulam-like recurrence pattern (see Fig. 1) where the phase trajectorty of the system periodically returns to the initial one corresponding, in an untrapped case, to a homogeneous state.

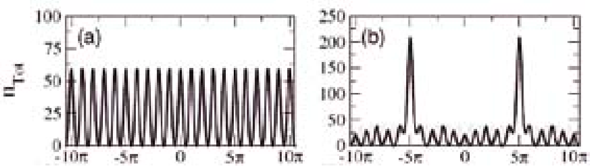


Figure 2: Total density of a polar ($^{23}$Na) spinor condensate prepared as a Bloch state at the edge of the first Brillouin zone of a 1D optical lattice of moderate depth (of $4$ recoil energies) shown at (a) $t = 0$ and (b) after$t = 14$ ms of evolution at the band edge.

This general scenario persists in the presence of more complex spatial potentials, such as a periodic optical lattice. Our modeling of the nonlinear behaviour of spin-1 BECs with repulsive spin-independent interactions and either ferromagnetic or anti-ferromagnetic (polar) spin-dependent interactions, loaded into a 1D optical lattice potential [2] revealed that dynamical instabilities in both types of spinors leads to the formation of nonlinearly localized multi-component structures (as seen in Fig. 2), with the overall density displaying periodically recurring, irregular arrays of spatial solitons.

## References

[1] E.V. Doktorov, V.M. Rothos, and Yu.S. Kivshar, Phys. Rev. A **76**, 013626 (2007).

[2] B.J. Dąbrowska-Wüster, E.A. Ostrovskaya, T.J. Alexander, and Yu.S. Kivshar, Phys. Rev. A **75**, 023617 (2007).