Phase-space Representation for Qubits

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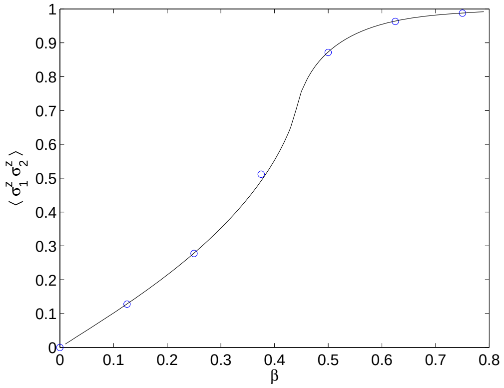
Problems involving interacting spins or qubits are often regarded as computationally intractable. These are frequently considered as only being accessible using quantum computers, which are not yet developed. At the same time, there is a ‘chicken and egg’ problem: it is difficult to design a quantum computer with no effective means to simulate its behaviour, including inevitable sources of loss and decoherence. In an effort to provide an avenue towards computational means to treat such problems, we have introduced a phase-space representation for qubits and spin models.

The technique uses an SU(n) coherent-state basis and can equally be used for either static or dynamical simulations. We review previously known definitions and operator identities, and show how these can be used to define an off-diagonal, positive phase-space representation analogous to the positive-P function. As an illustration of the phase-space method, we use the example of the Ising model, which has exact solutions for the finite-temperature canonical ensemble in two dimensions. We show how a canonical ensemble for an Ising model of arbitrary structure can be efficiently simulated using SU(2) or atomic coherent states. The technique utilizes a transformation from a canonical (imaginary-time) weighted simulation to an equivalent unweighted real-time simulation.



In the paper [1], the results are compared to the exactly soluble two-dimensional case. This is an important comparison, since this is the only known exactly soluble problem involving interacting spins in higher dimensions. The comparison graph shows how the phase-space simulation method compares with an exact solution, in calculating spin correlation functions. We see that there is excellent agreement even at the critical temperature. This provides evidence that the new technique gives correct results, in a case involving strong correlations and fluctuations near a phase transition.

We note that Ising models in one, two, or three dimensions are potentially achievable experimentally as a lattice gas of ultracold atoms in optical lattices. The technique is not restricted to canonical ensembles or to Ising-like couplings. It is also able to be used for real-time evolution and for systems whose time evolution follows a master equation describing decoherence and coupling to external reservoirs. The case of SU(n) phase space is used to describe n-level systems. In general, the requirement that time evolution be stochastic corresponds to a restriction to Hamiltonians and master equations that are quadratic in the group generators or generalized spin operators.

In future, we hope to develop this technique further, with a view towards treating useful problems in quantum information theory.

References

[1] D. Barry and P. D. Drummond, Phys. Rev. A **78**, 052108 (2008).