Number Phase Wigner Representation for Efficient Stochastic Simulation

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Phase-space representations based on coherent states (P, Q, Wigner) have been successful in the creation of stochastic differential equations (SDEs) for the efficient stochastic simulation of high-dimensional quantum systems [1]. By combining the equation of motion produced by these representations with a Fokker-Planck equation, one can produce a set of stochastic differential equations (SDEs) that can greatly reduce the dimensionality of a problem. For example, consider a BEC modelled as a set of N harmonic oscillators. This would typically require a density matrix with D^{2N} components to solve directly (where D is the number of elements you have in your truncated basis). Using a phase space representation one can change this to a set of only N SDEs. This allows quantum simulations on a desktop computer that are otherwise unfeasibly large on any imaginable classical computer.

Unfortunately, the high nonlinearities in a Bose Einstein Condensate (BEC) make it extremely difficult to simulate for long times without making a semiclassical approximation. Long-term simulation of the full dynamics of the quantum field is important for understanding experiments where mean field and perturbative methods fail, such as the dynamics of an atom laser under feedback control. The nonlinearities are the dominant term in the dynamics, and preserve atom number, but they do not preserve a coherent state. We therefore hypothesise that representations based on coherent states are inappropriate for these systems, and that a number-phase based alternative will do better. Historically the investigation of number-phase space methods have been primarily concerned with the visualisation of quantum states and no simple extension can be used to generate SDEs. We presented a novel number-phase Wigner representation that does generate SDEs. We investigated the properties of this new distribution and used it to efficiently produce some potentially useful numerical results.

We examined a single mode problem of a damped anharmonic oscillator, which is analogous to a single-mode BEC. We simulated this system using three different scalable phase space methods: our novel number-phase Wigner representation, coherent truncated Wigner and positive-P. The novel number-phase method required a approximate truncation in some terms, similar to the coherent truncated Wigner method. These solutions were compared to an analytic solution as shown in figure 1. We found the number-phase Wigner representation converged for a longer time than any other competing method and was even able to reconstruct the dampened phase revival of the initial coherent state. These results show this representation has great potential for quantum field simulations where number or phase-conserving terms are dominant, such as BEC. We plan to extend this method to multimode problems, such as the quantum dynamics of a feedback-controlled atom laser.

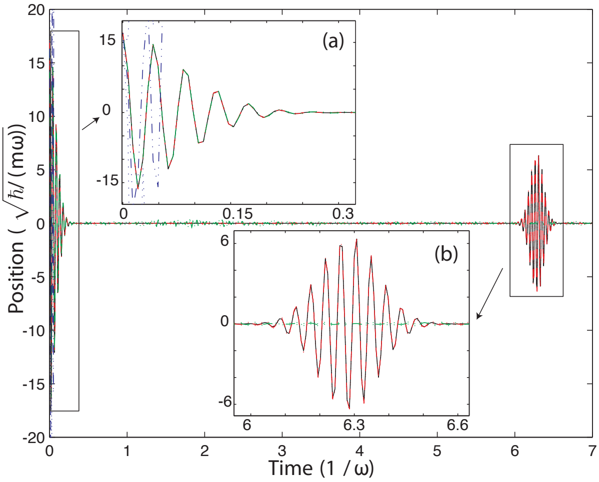

Fig. 1: Position vs. time for a damped anharmonic oscillator integrated using the number-phase Wigner representation (red), truncated Wigner representation (green), gauge-P^+ (blue) and analytic solution (black).

References

[1] C.W. Gardiner, Handbook of stochastic Methods, Springer-Verlag (1983).