# Theory of two-component BEC interferometry

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Many previous treatments [1, 2] for two component BEC interferometry are based on the simplest assumption, namely that during the interferometric process the condensate is unfragmented, with all bosons occupying the same single particle state. The latter is a linear superposition of the two internal states. Equations for the spatial wave functions associated with these internal states have been obtained in the form of coupled Gross-Pitaevskii equations. However, there are two distinct single particle states each boson could occupy, and for $N$ bosons the $N+1$ dimensional state space for such two mode theories - such as the present theory and similar work in [3, 4] - allow for more general quantum states than just those that are unfragmented. Two mode theories have previously been developed for single component BECs with two orthogonal spatial modes (such as in double-well interferometry) [5, 6, 7], and fragmentation effects shown in [7].

The quantum state of the $N$ boson system is written as a superposition of the Fock states with amplitudes $b_k(t)$. Each Fock state is a fragmented state, with definite numbers $\frac{N}{2} \mp k$ of bosons respectively in the two modes $\phi_F(\mathbf{r},t)\,|\,F\rangle$ and $\phi_G(\mathbf{r},t)\,|\,G\rangle$ (where the internal states are $|F\rangle$, $|G\rangle$ and the spatial mode functions are $\phi_F(\mathbf{r},t)$, $\phi_G(\mathbf{r},t)$). The Dirac-Frenkel variational principle can be used to obtain matrix mechanics equations for the amplitudes $(k = -N/2, .., N/2)$

$$i\hbar \frac{\partial b_k}{\partial t} = \sum_l (H_{kl} - \hbar U_{kl}) b_l$$

and generalized Gross-Pitaevskii equations for the mode functions $(a = F, G)$

$$X_{aa}\, i\hbar \frac{\partial}{\partial t}\phi_a = X_{aa}(-\frac{\hbar^2}{2m}\nabla^2 + V_a)\,\phi_a + \sum_{b\neq a} X_{ab}\,\Lambda_{ab}\,\phi_b + \sum_b (g_{ab} Y_{ab\,ba}\,\phi_b^*\,\phi_b)\,\phi_a.$$

The $N+1$ amplitude equations describe the system evolution amongst the possible Fock states. The Fock state Hamiltonian and rotation matrix elements $H_{kl}$, $U_{kl}$ depend on the mode functions $\phi_a(\mathbf{r}, t)$. The two coupled Gross-Pitaevskii equations are non-linear in the mode functions. The one and two body correlation functions $X_{ab}$ and $Y_{ab\,ba}$ depend quadratically on the amplitudes $b_k(t)$, and reflect the relative importance of the different Fock states during the interference process. The trap potential is $V_a$, the inter-component coupling term is $\Lambda_{ab}$ and collisions are described by the $g_{ab}$.

The present self-consistent amplitude and mode equations are more general than expressions in [1,2], and differ from those in [3,4]. They can be used to treat various BEC interferometry experiments, such as Ramsey interferometry. Collisions may be ignored during the short coupling pulses, where the evolution is shown to be equivalent to rotations of the Bloch vector. During the non coupling evolution, the magnitudes $|b_k(t)|$ of the amplitudes remain constant, but there is evolution of the phase factors $A_k(t)$, where $b_k(t) = |b_k(t)| \exp(-iA_k(t)/\hbar)$. Fragmentation effects would imply that the Bloch vector no longer remains on the Bloch sphere.

## References

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