Recent Applications of Classical Field Theory

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Overview

- 0. PGPE Review
- I. Application to (quasi)-2D trapped system
- II. Application to trapped critical system

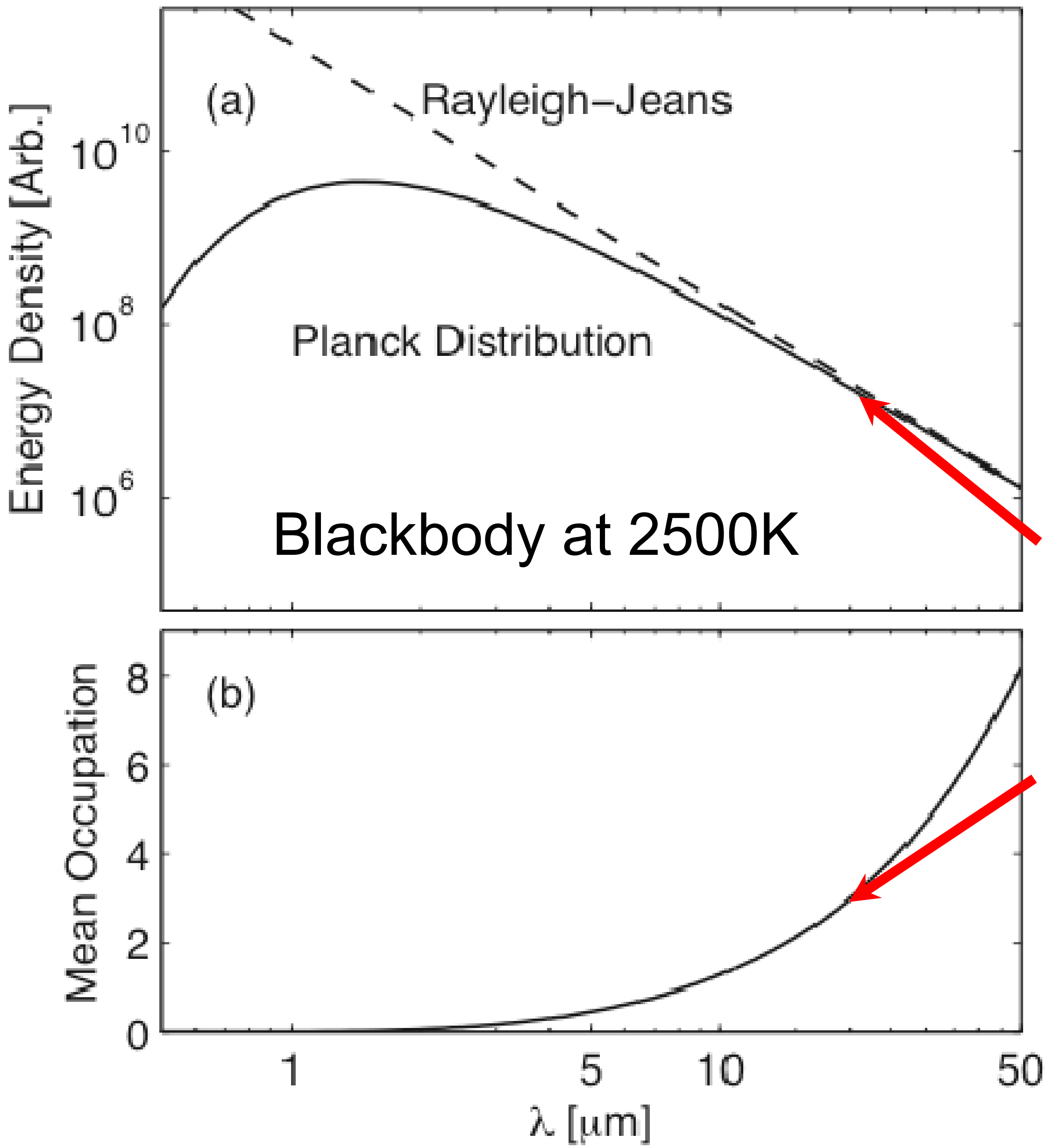
0.
PGPE Review

(*Projected Gross-Pitaevskii Equation*)

Main Objective:
Dynamical Finite-T Theory

- Non-perturbative
- Flexible and computationally tractable as GPE
- Applications: Condensate growth, Vortex lattice formation, Atom lasers, Strongly fluctuating systems (near Tc or low dimensional).
- Quantitative comparison with experiment

Motivation: Blackbody Spectrum



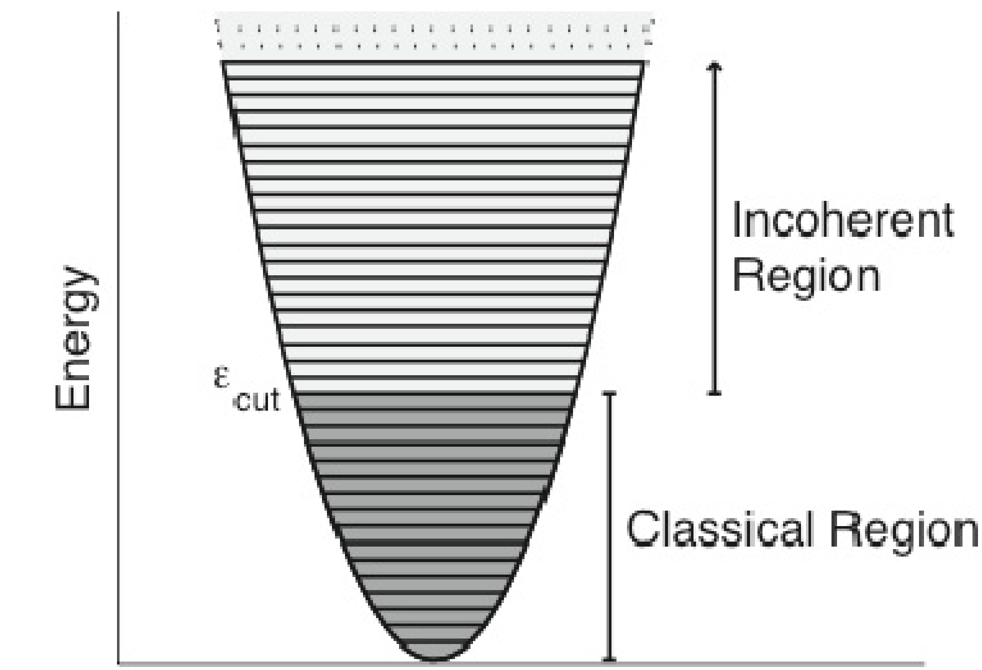
>3 photons per mode:classical field well-approximates quantum field

Classical Region for Atoms

Conservation of particles $\rightarrow \mu$

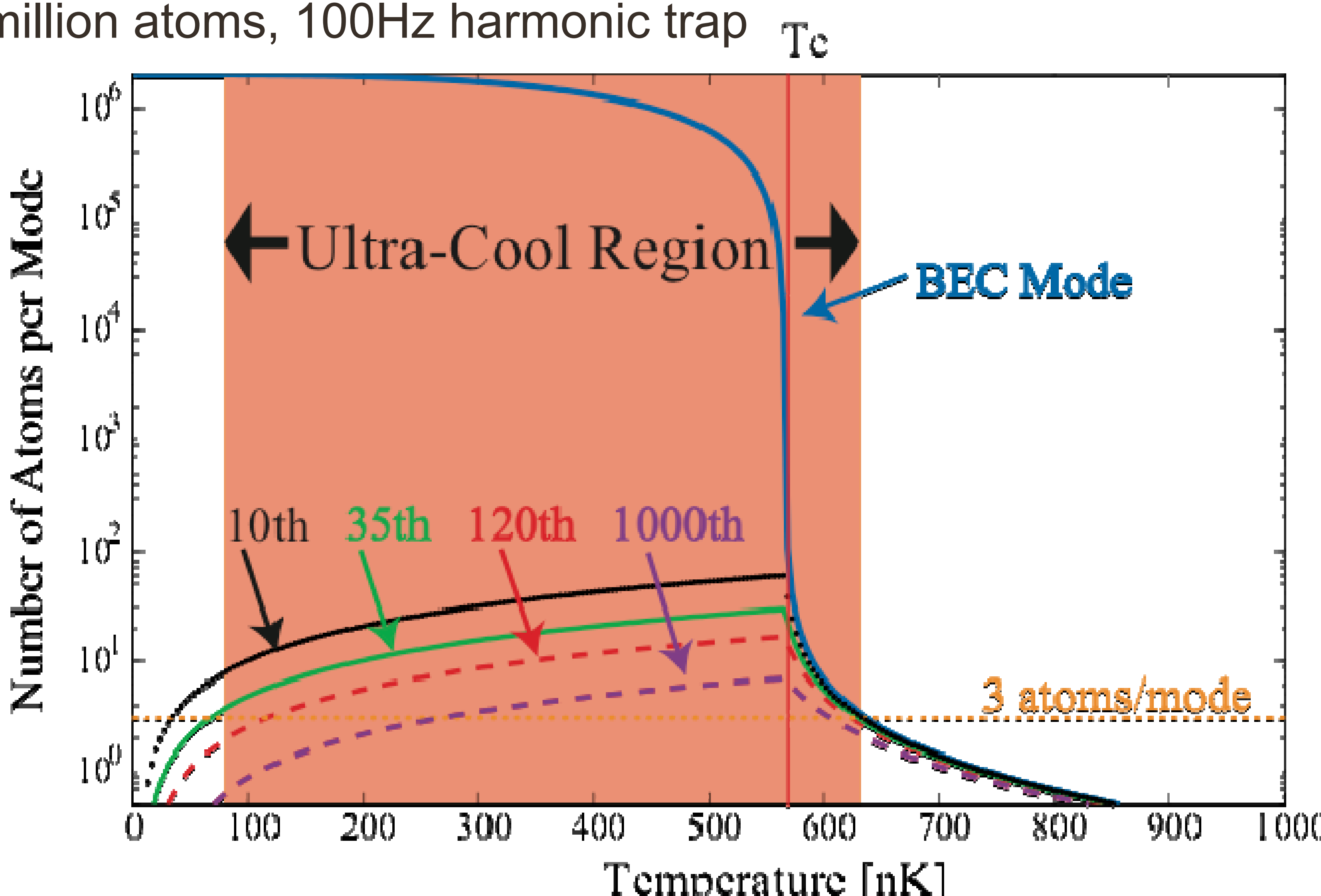
Classical regime: $\varepsilon_n - \mu \lesssim kT$

Occurs only at extremely low temperatures

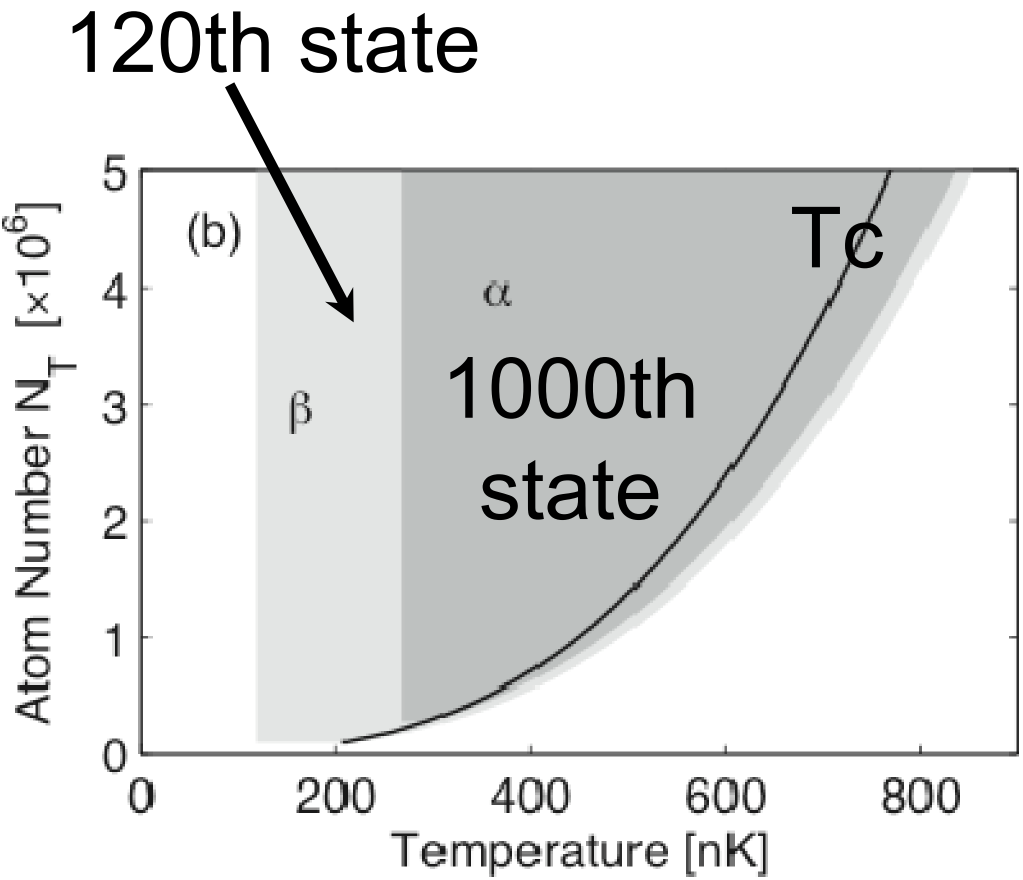


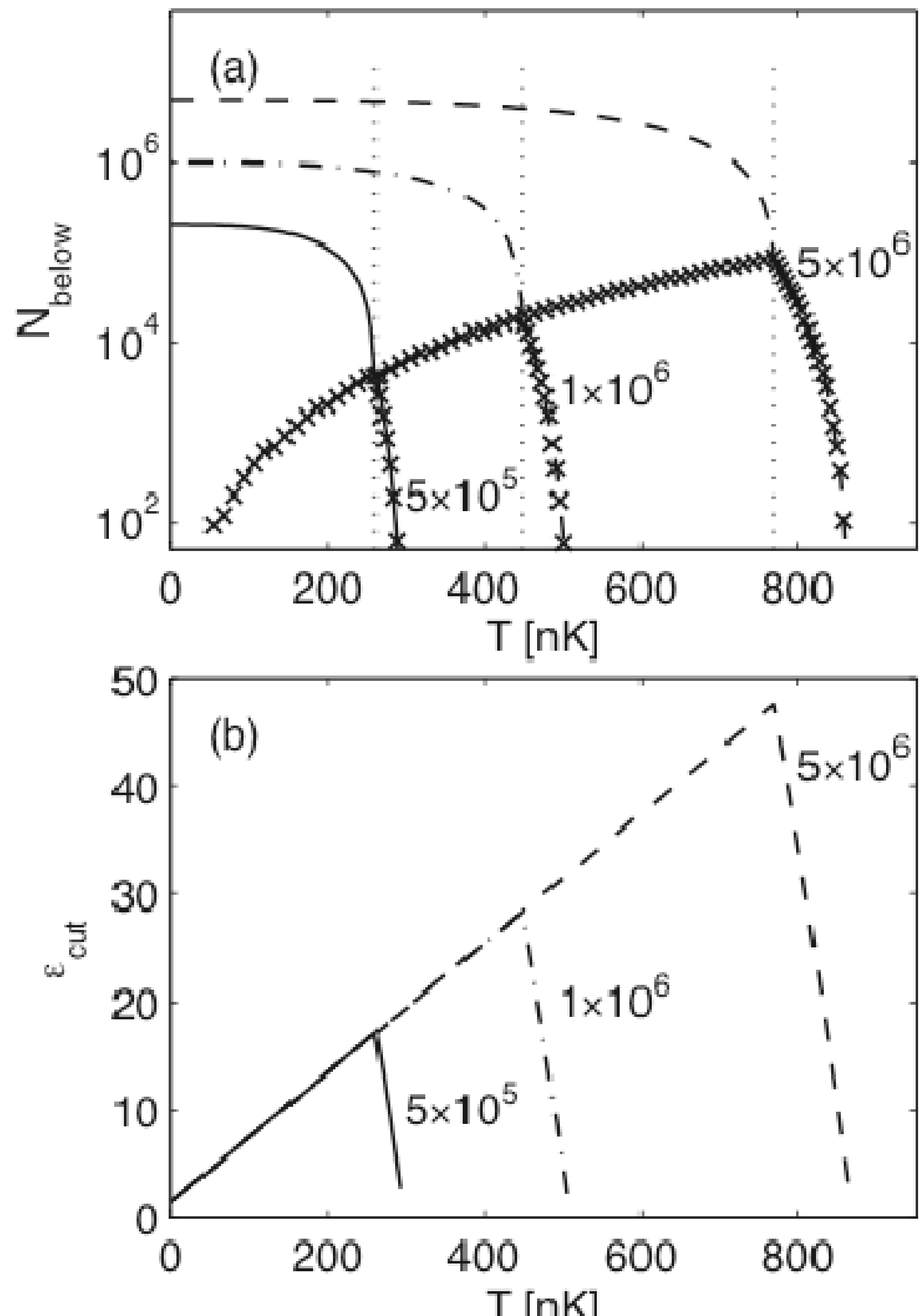
Ideal Bose Gas

2 million atoms, 100Hz harmonic trap



Classical Region





PGPE Formalism

(EOM for the classical region)

The Projected Gross-Pitaevskii Equation:

projector

$$i\hbar \frac{\partial \Psi_C}{\partial t} = \hat{H}_{sp}\Psi_C + \mathcal{P}\{U_0|\Psi_C|^2\Psi_C\}$$

restricted to classical region

single particle modes: Eigenstates of $\hat{H}_{sp}$

$$\Psi_C = \sum_{j=1}^{n} c_j \phi_j(\mathbf{x})$$

Harmonic trap: $\phi_j(\mathbf{x})$ **harmonic oscillator states**

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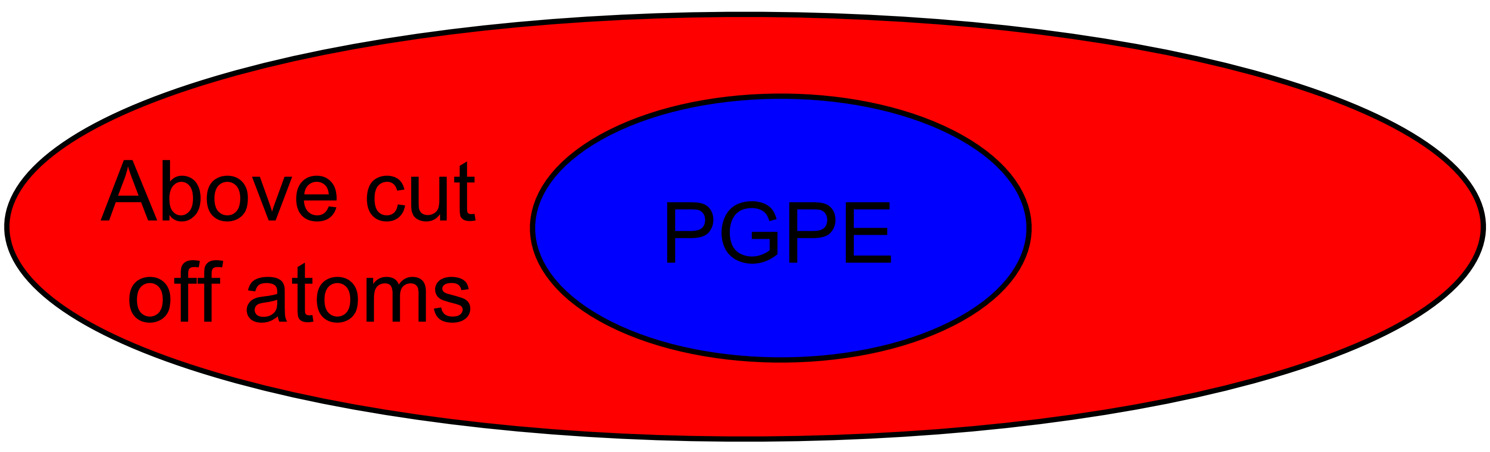
Above cut off atoms?

- Treat as weakly coupled system (thermal and diffusive equilibrium)
- Describe using semiclassical meanfield theory

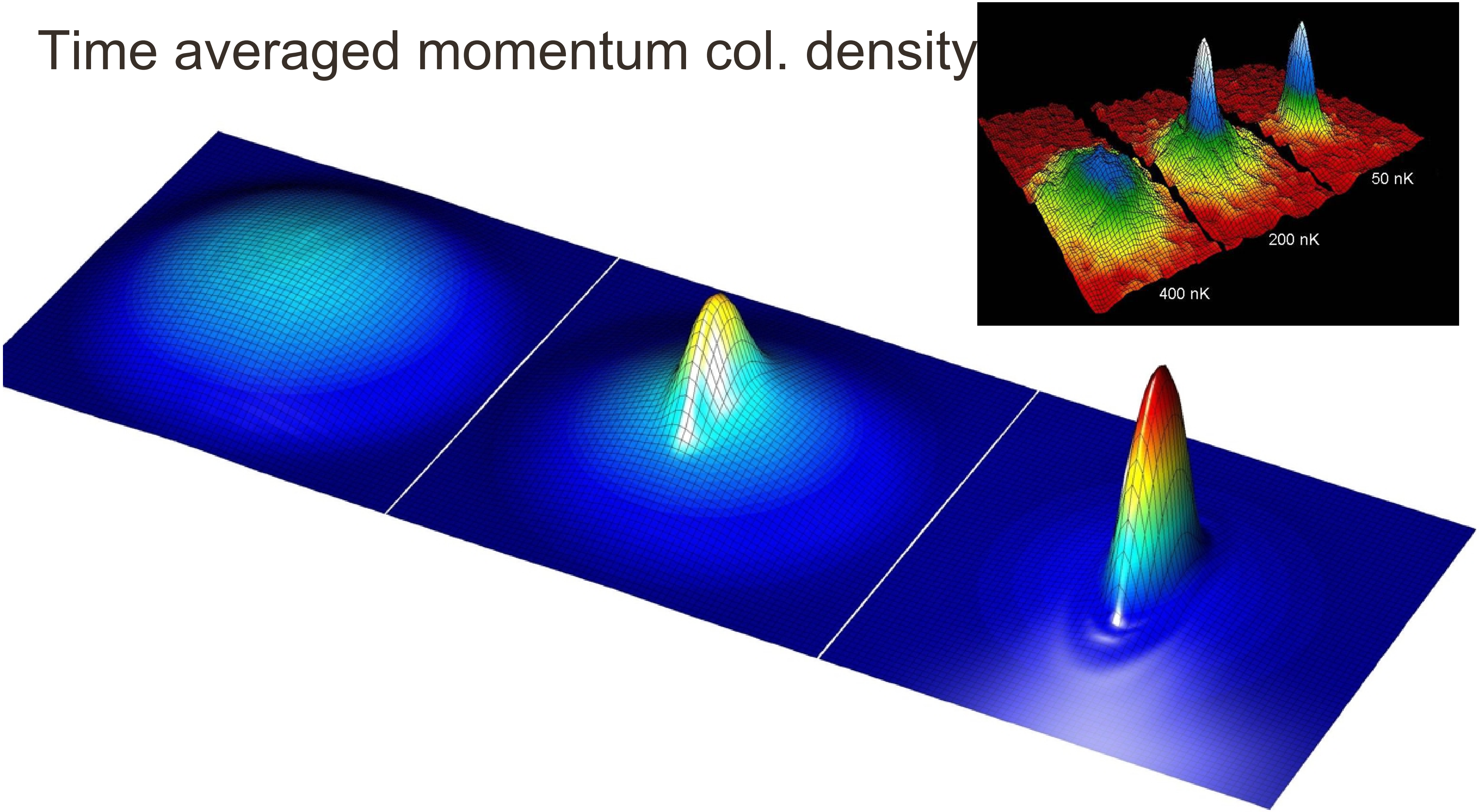


Other Details

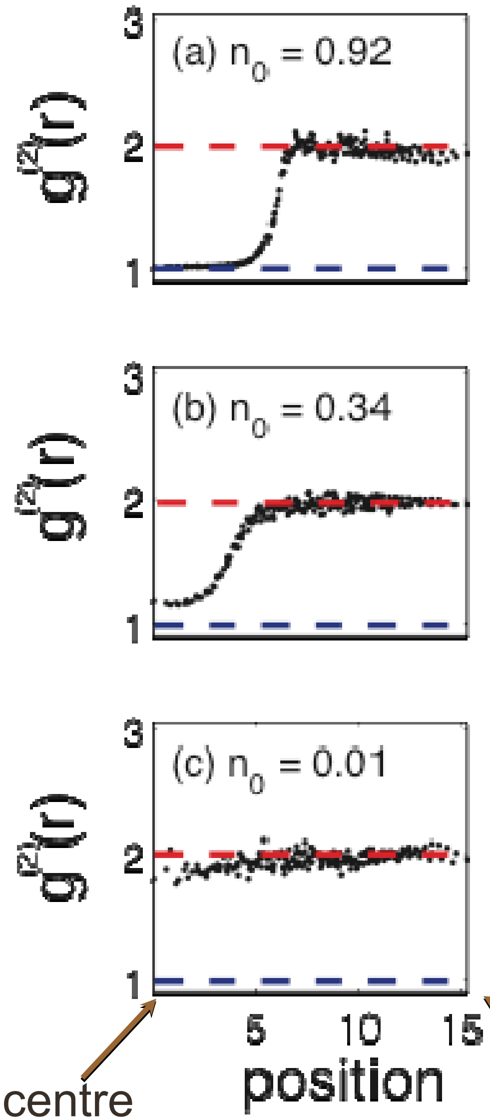
- Equilibrium: Ergodicity
- Techniques to extract temperature, chemical potential and condensate fraction

Condensation

Time averaged momentum col. density

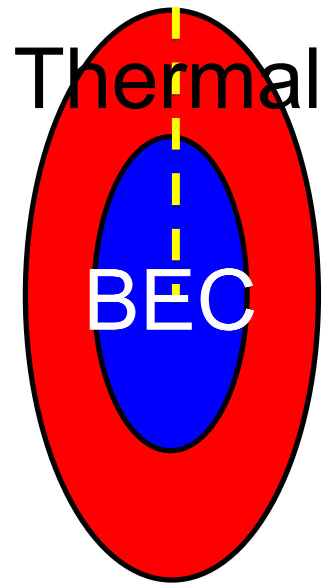


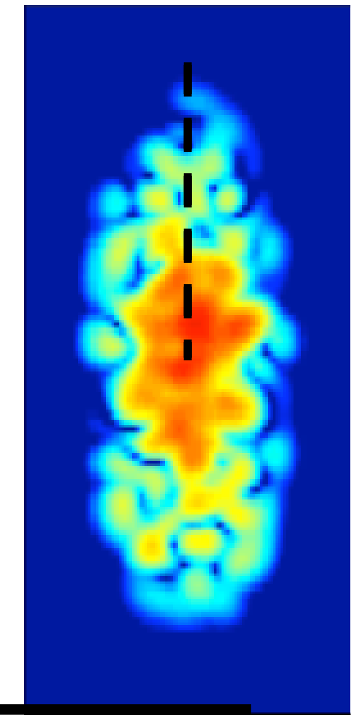
Fluctuations



trap centre

cloud edge





Density Fluctuations:

$$g^{(2)}(\mathbf{r}) = \frac{\langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle}{\langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle^2}$$

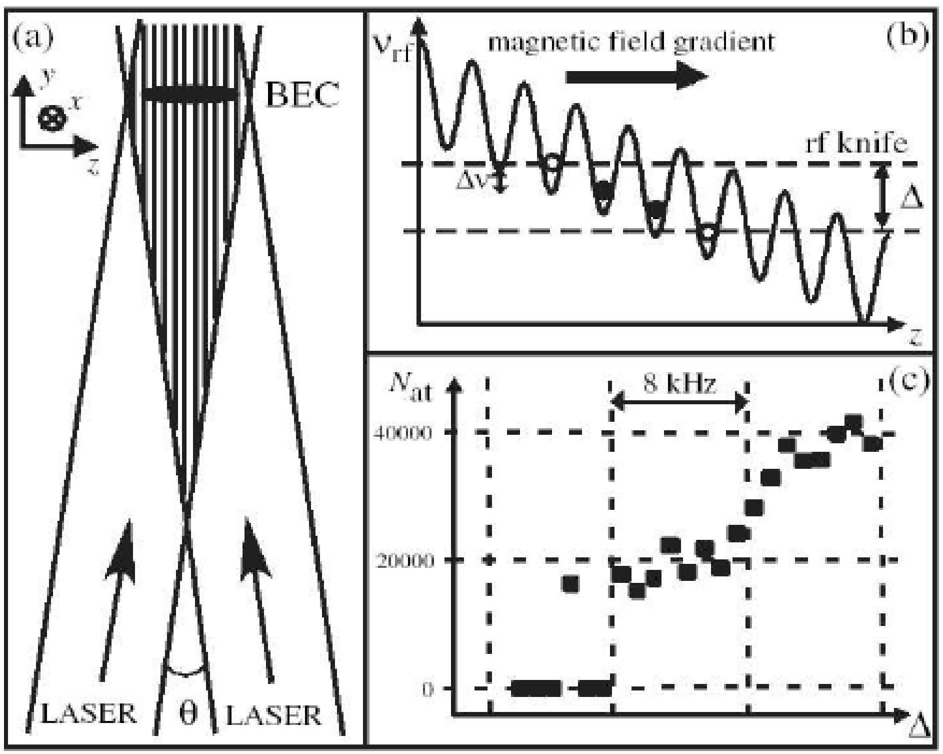
I.
Application to 2D Trapped Systems

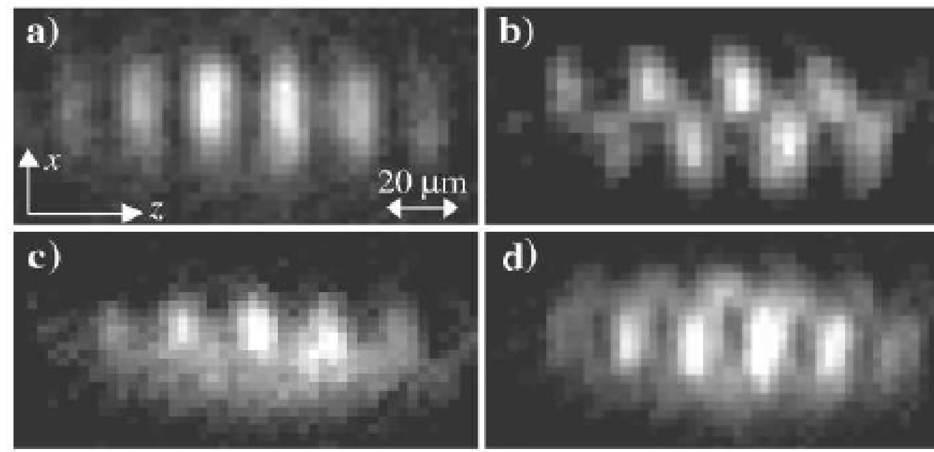
2D Bose Gas

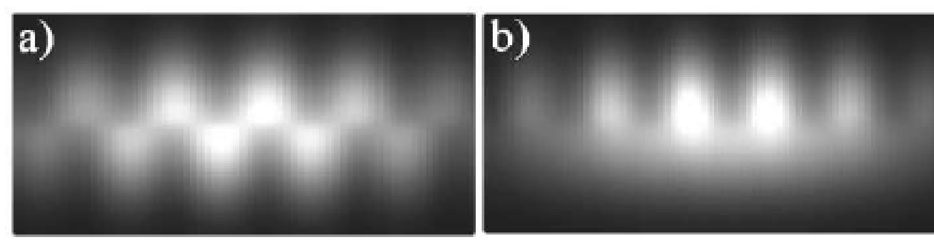
Fundamental question:

What is the nature of the low temperature superfluid phase(s) of the 2D trapped Bose gas?

Observation of Phase Defects in Quasi-Two-Dimensional Bose-Einstein Condensates[a]





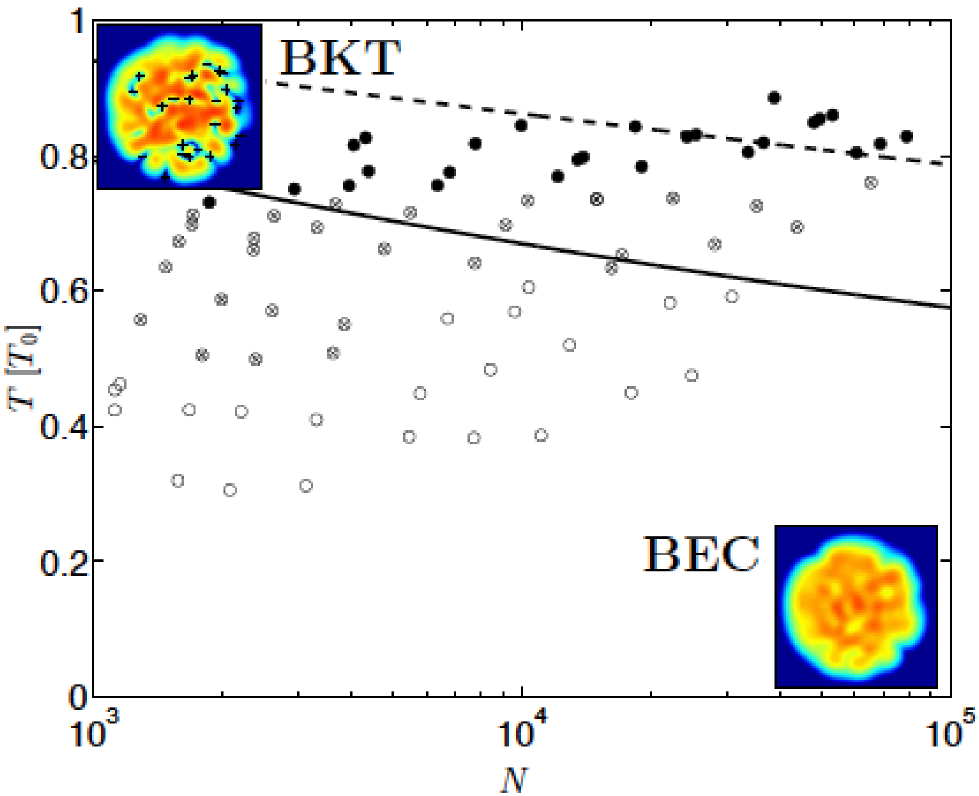


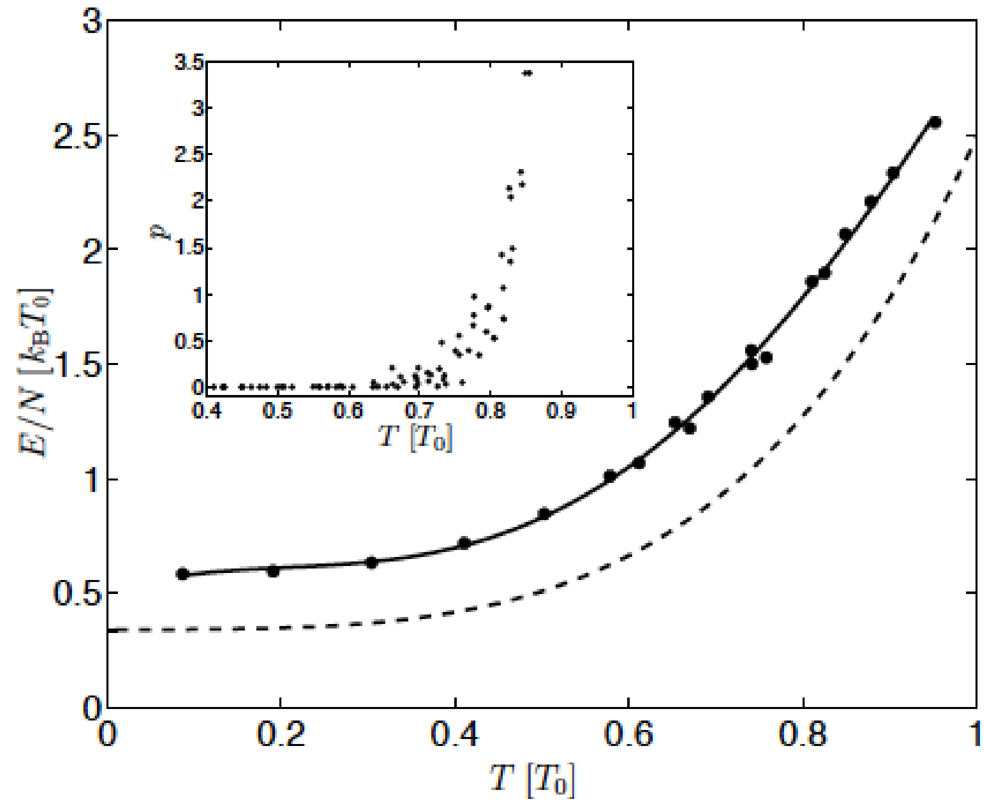
[a]S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau and J. Dalibard, *Phys. Rev. Lett.* **95**, 190403 (2005).

Experimental Issues

- Temperature unclear 0.7*Tc* -0.9*Tc* (?)
- Vortex pairs unobservable (?)

Classical field simulations 1/3[a]





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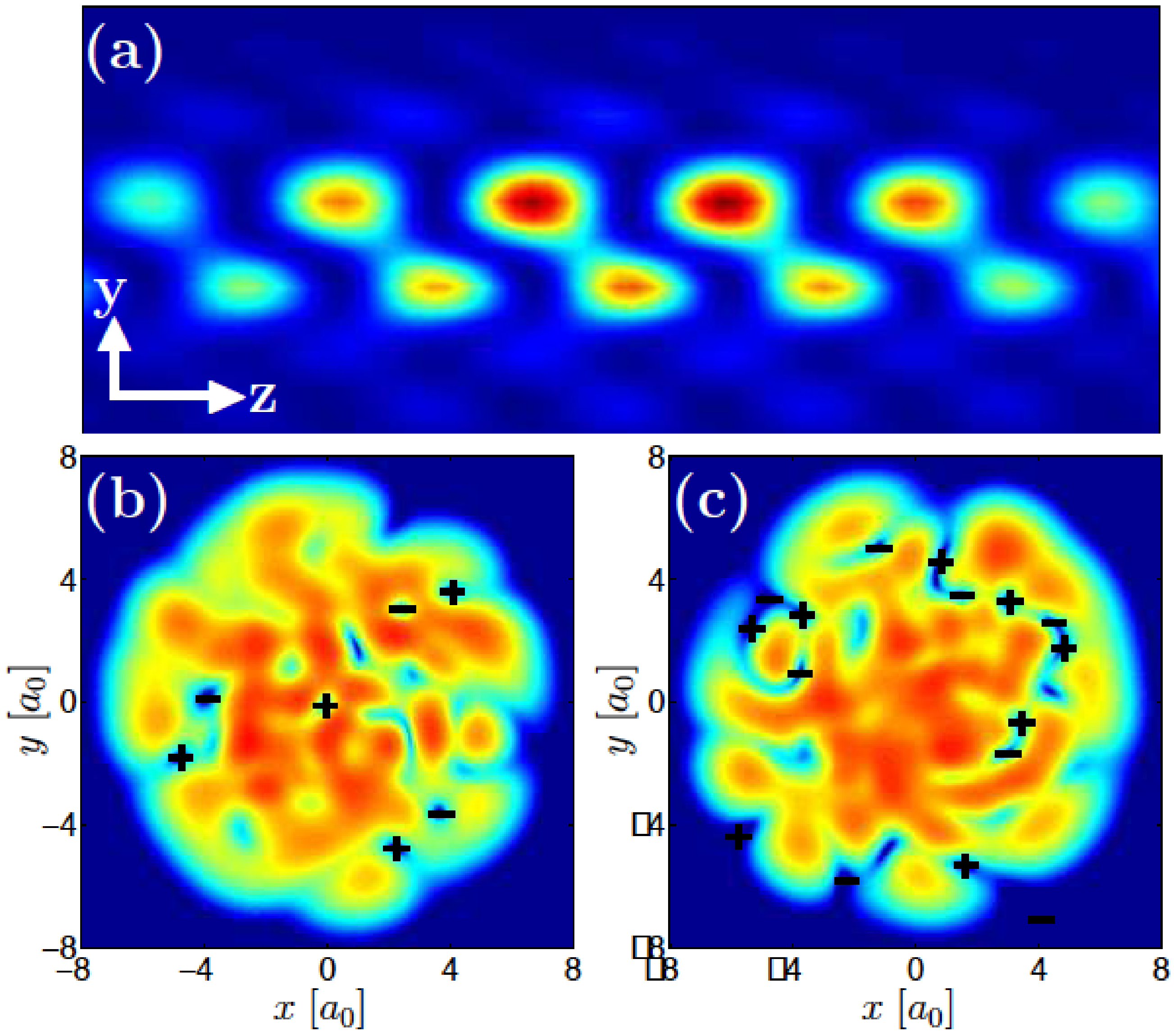
Classical field simulations 2/3



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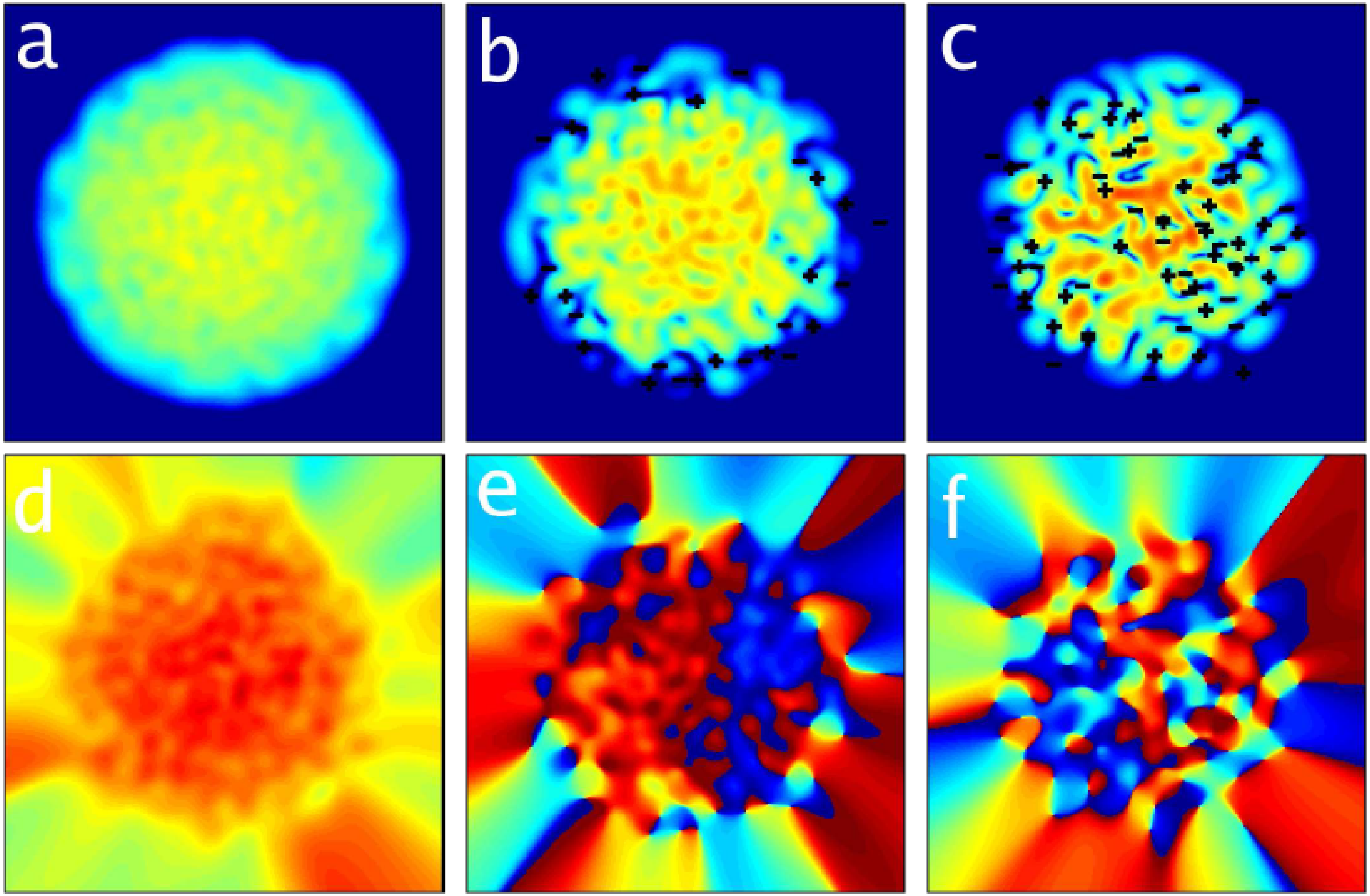
Classical field simulations 3/3



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Temperature Movie

QuickTime™ and a
decompressor
are needed to see this picture.

Current Work

- Phase diagram - understanding the BKT window
- Looking at experimental diagnostics: scissor modes, 3-body loss

II.
Application to Systems Near Tc

Interacting Bose Gas Critical Temperature

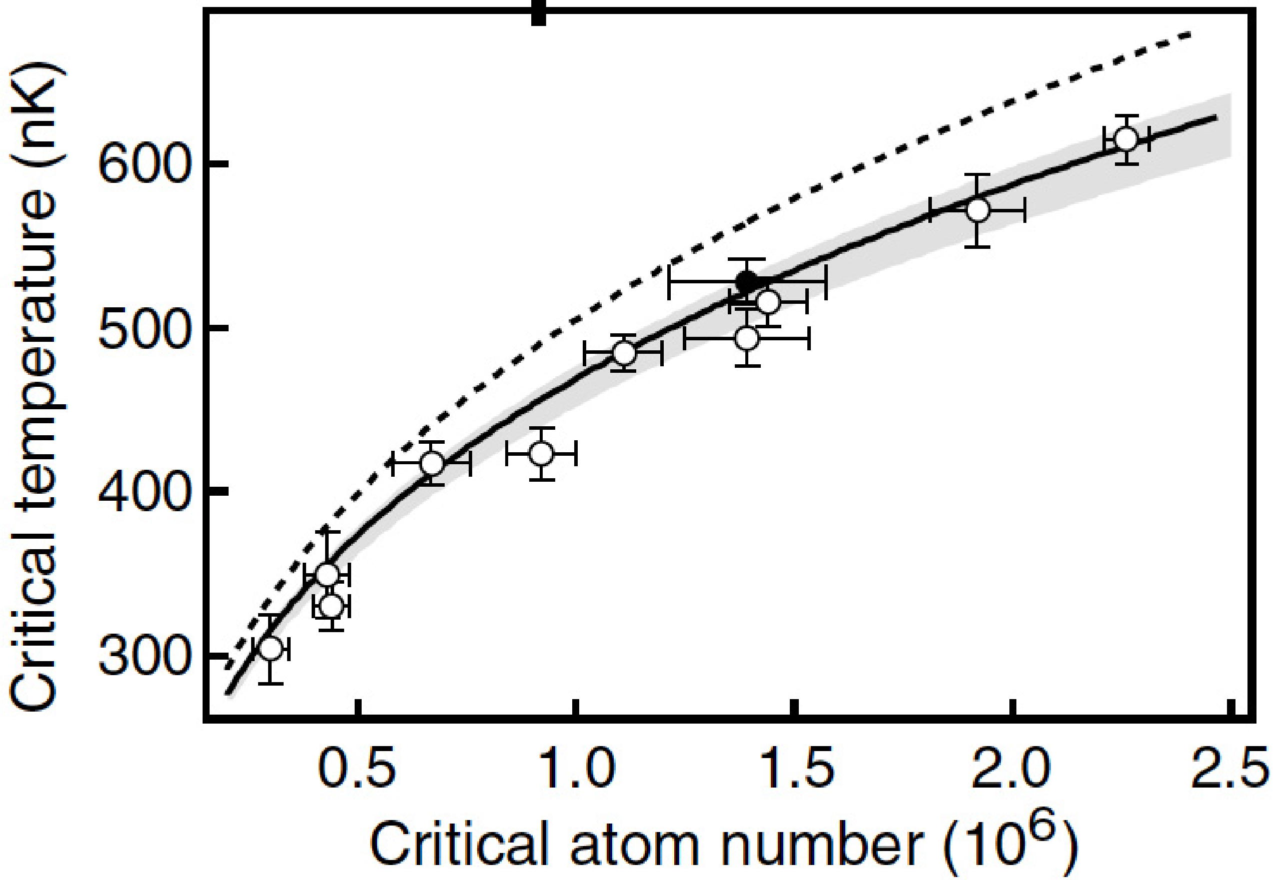
- Homogeneous gas, long history: Lee and Yang 1957, finally sorted ~2001.

$$\Delta T_c / T_c^0 = c a n^{1/3}$$

c = 1.4 ... PGPE gets this right!!
(Davis et al., 2003)

- Inhomogeneous gas ?

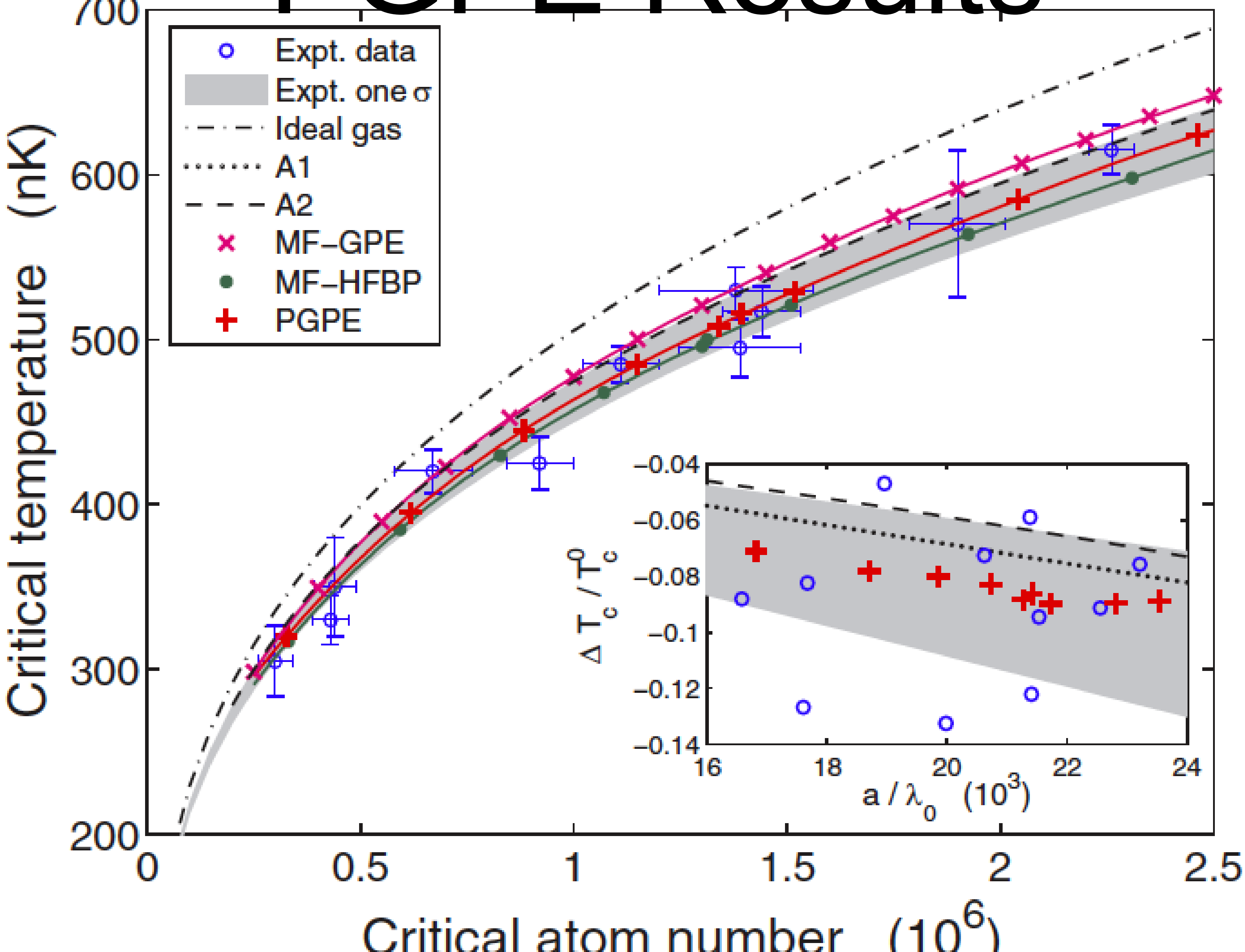
Experiments



- 50-to-1 cigar trap

Gerber *et al.,* PRL **93,** 030405 (2004)

PGPE Results



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summary

- Quantitative agreement between PGPE and experiment
- Quantitative calculations of critical effects in the trapped Bose gas
- currently: looking for regimes to enhance critical effects

Conclusions & Outlook

- Presented theoretical results for low dimensional and critical systems.
- Good agreement with experiment
- *future:* Correlations functions in the critical region and for quasi-low dimensional gases
- *future:* Non-equilibrium phenomenon

the end

References

- Gerber *et al.,* PRL **93,** 030405 (2005)
- P. B. Blakie and M. J. Davis, PRA **72**, 063608 (2005)
- Stock *et al.,* PRL **95,** 190403 (2005)
- T. Simula and P.B. Blakie PRL **96,** 020404 (2006)
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