Ultracold Molecule Production With Resonant Oscillations of a Magnetic Field

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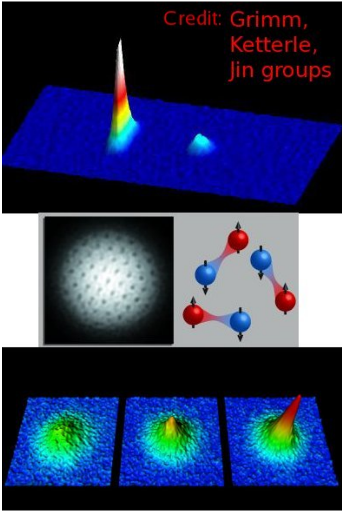
Making Molecules



Motivations:

- Superfluidity
- BCS-BEC crossover
- Collisions
- Bright solitons
- Heteronuclear molecules

Making Molecules:

- Photoassociation
- Feshbach resonances:
  - Sweeps
  - Strong interactions near resonance
  - Ramsey interferometry

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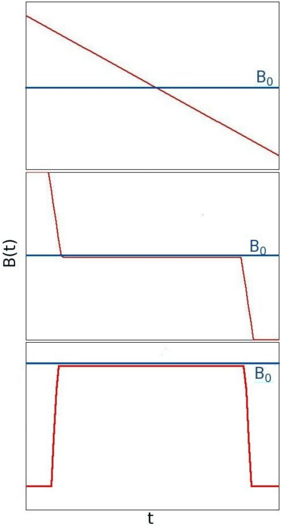

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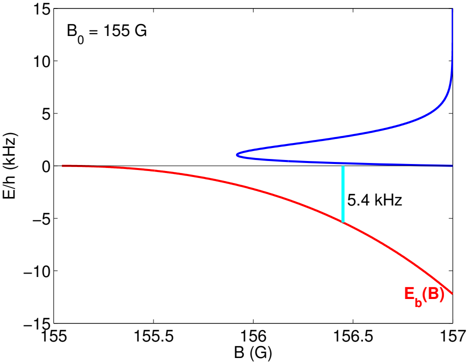
kHz Energies



- Molecular binding energies $E_{\mathrm{b}}(B)$ of order kHz.
- Thermal energy $k_{\mathrm{B}} \times (20 - 80)$ nK $\approx h \times (0.4 - 1.6)$ kHz.
- Interference effects are on ms timescales

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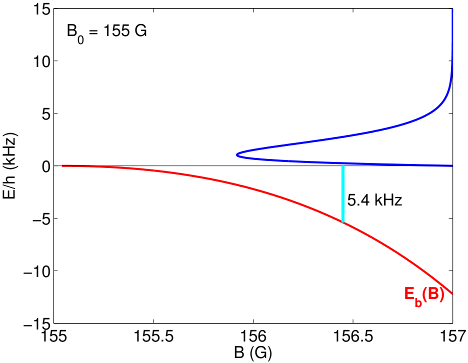

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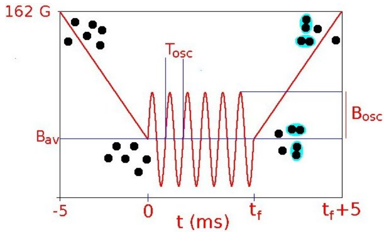

- ^{85}Rb, 155 G resonance.
- Non-degenerate and partially condensed gases.
- RF pulse duration $\leq$ 40 ms.

- Damped oscillations at short times.
- Maximum $\approx 30\,\%$.
- Lorentzian lineshape, linewidth increasing with pulse duration.
- Temperature dependent shift in resonant frequency.
- Maximum, minimum, revival as oscillation amplitude is increased (private communication).

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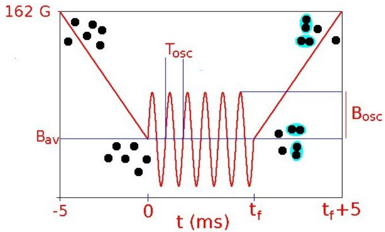

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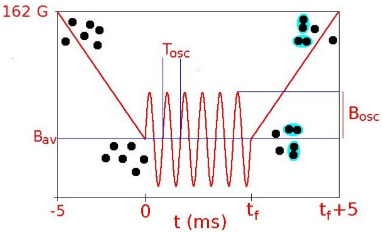

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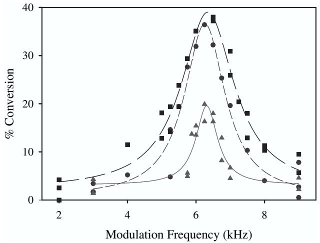

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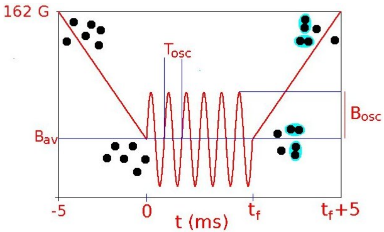

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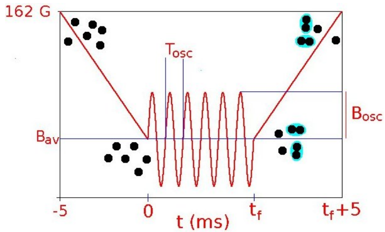

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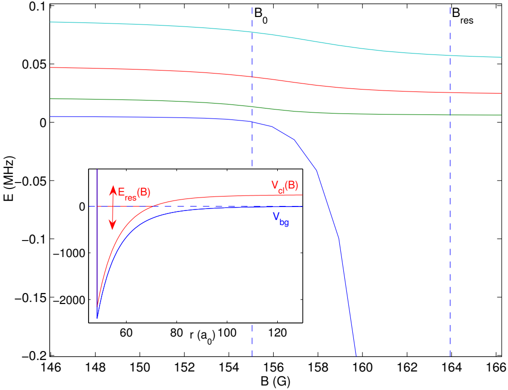
Two Channel Model



Two channel Hamiltonian:

$$H_{2B} = \begin{pmatrix} H_{ent} & W \\ W & H_{cl} \end{pmatrix}$$

Closed channel population entirely in resonance state ϕ_{res}:

$$H_{cl} = |\phi_{res}\rangle E_{res}(B) \langle\phi_{res}|$$
$$E_{res}(B(t)) = \frac{\partial E_{res}}{\partial B}(B(t) - B_{res})$$
$$B(t) = B_{av} + B_{osc}\sin(\omega_{osc} t)$$

Transition Probability ⇒ Conversion Efficiency

- The continuum is represented using a set of scattering states at the average magnetic field B_{av}, $\{|\phi_{\mathbf{p}}^{\mathrm{av}}\rangle\}$.
- We calculate $\langle\phi_{\mathrm{b}}^{\mathrm{av}}|U_{2\mathrm{B}}(t,0)|\phi_{\mathbf{p}}^{\mathrm{av}}\rangle$, where $|\phi_{\mathrm{b}}^{\mathrm{av}}\rangle$ is the bound state at B_{av}, and $U_{2\mathrm{B}}(t,0)$ is the two-body evolution operator.
- The conversion efficiency at t is given by the thermal average over the transition probability density $\rho(p,t) = \left|\langle\phi_{\mathrm{b}}^{\mathrm{av}}|U_{2\mathrm{B}}(t,0)|\phi_{\mathbf{p}}^{\mathrm{av}}\rangle\right|^2$:

$$\frac{N_{\mathrm{mol}}}{N}(t) = n(2\pi\hbar)^3\left(\frac{\beta}{\pi m}\right)^{3/2}\int d\mathbf{p}\exp\left(\frac{-\beta p^2}{m}\right)\rho(p,t)$$

where n is the gas density, $\beta = 1/k_{\mathrm{B}}T$, and m is the atomic mass.

- Assumption: Thermal gas treated as reservoir.

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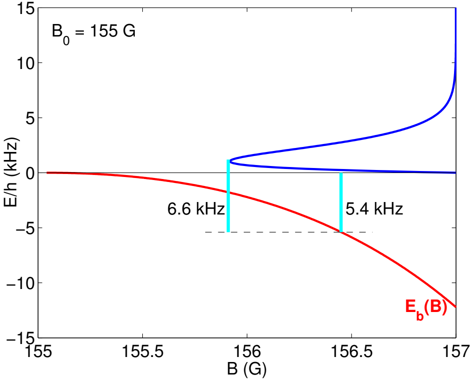
Resonance condition



- Resonance condition: $E_{\mathrm{b}}^{\mathrm{av}} - \frac{p^2}{m} + \hbar\omega_{\mathrm{osc}} = 0$.
- Free atom pair relative kinetic energy p^2/m.
- Continuum makes physics different to two-level case, considered by Bertelsen and Mølmer (PRA **73**, 013811 (2006)).

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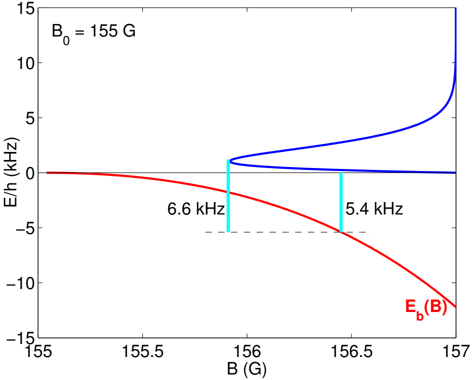

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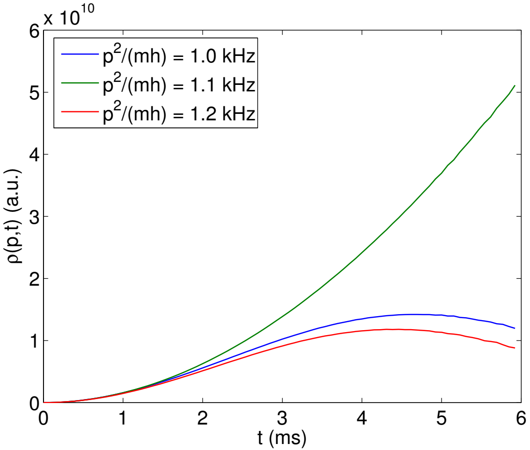
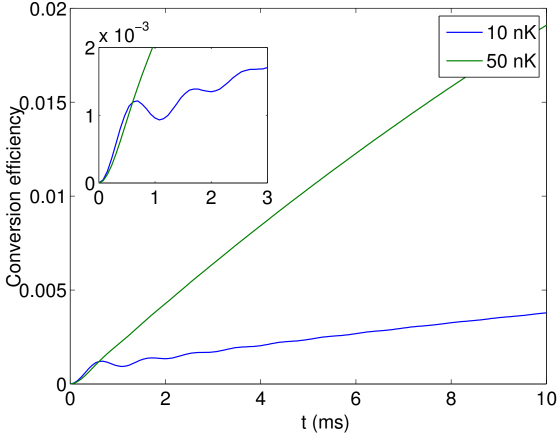
Resonant Growth in Transition Probability Density



For $B_{\mathrm{av}} = 156.45$ G,
$E_{\mathrm{b}}^{\mathrm{av}}/h = -5.4$ kHz,
$\omega_{\mathrm{osc}}/2\pi = 6.5$ kHz,

$E_{\mathrm{b}}^{\mathrm{av}} - \frac{p^2}{m} + \hbar\omega_{\mathrm{osc}} = 0$
for $p^2/mh = 1.1$ kHz
$\Rightarrow$ Resonant growth in $\rho(p,t)$

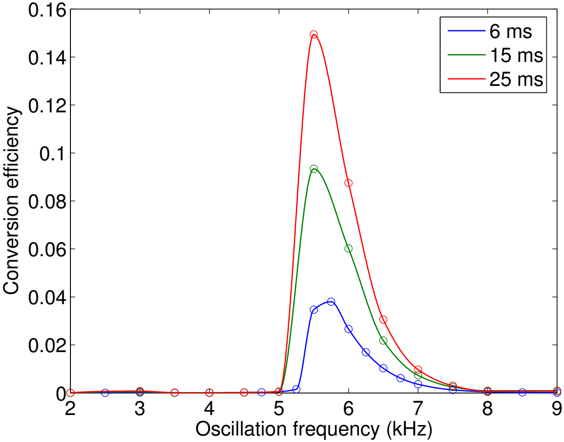
Conversion Efficiency: Time Dependence



$n = 1 \times 10^{11}$ /cm^3

Dephasing of continuum modes: Damped oscillations more distinct when the momentum distribution is narrower.

Conversion Efficiency: Frequency Dependence



- $T = 20$ nK, $n = 1 \times 10^{11}$ /cm^3.
- Almost no conversion for $\omega_{\text{osc}} < |E_{\text{b}}|/\hbar$ $\Longrightarrow$ Losses? Many-body?

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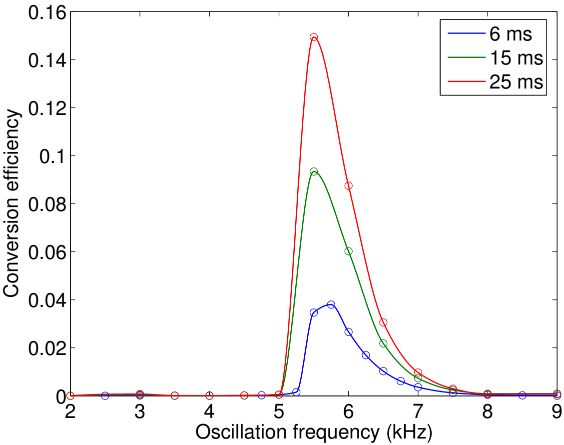

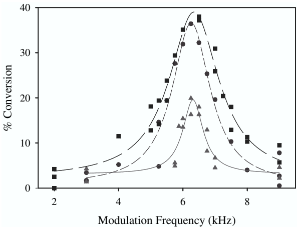

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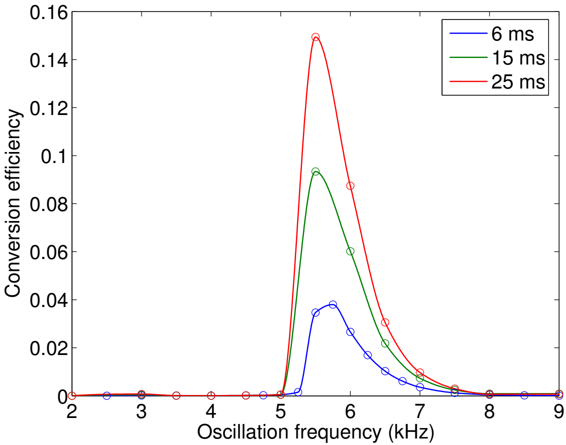

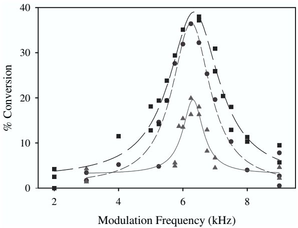

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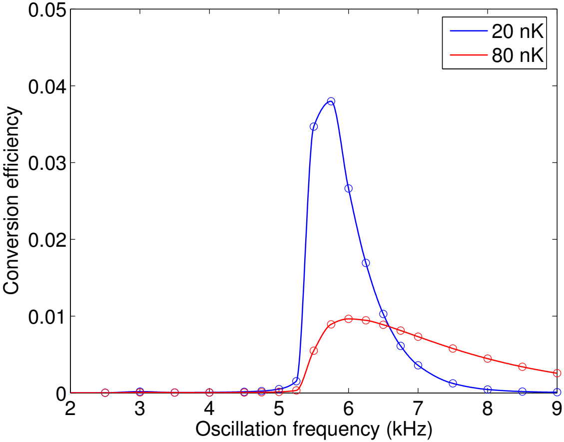

Wider momentum distribution at higher temperatures

$\Rightarrow$ conversion efficiency is less dependent on ω_{osc}

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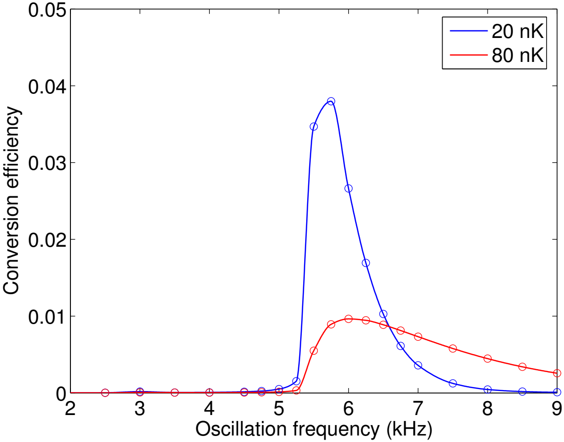

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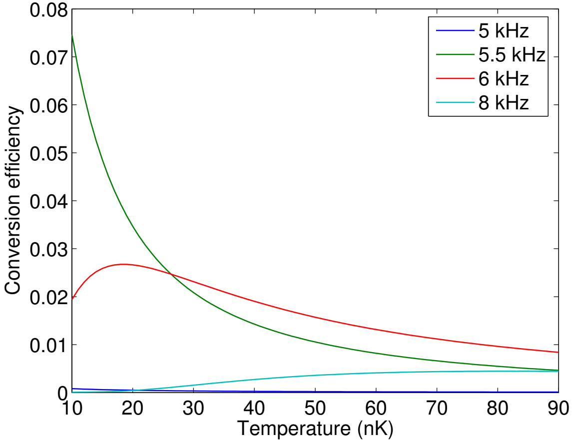
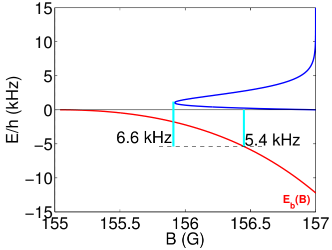
Conversion Efficiency: Temperature Dependence





Consider peak and width of thermal distribution

⇒ Match T and ω_{osc}.

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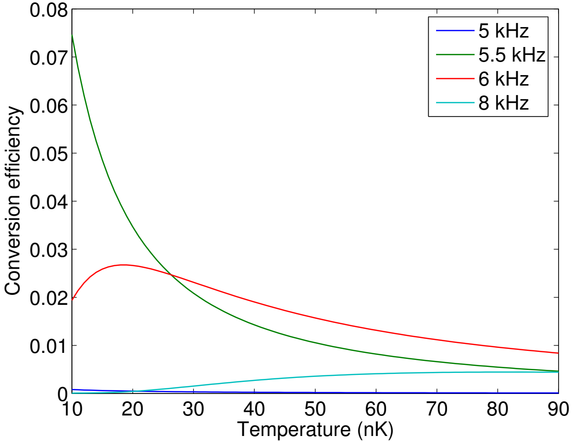

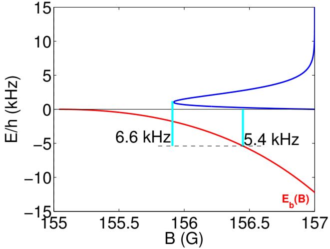

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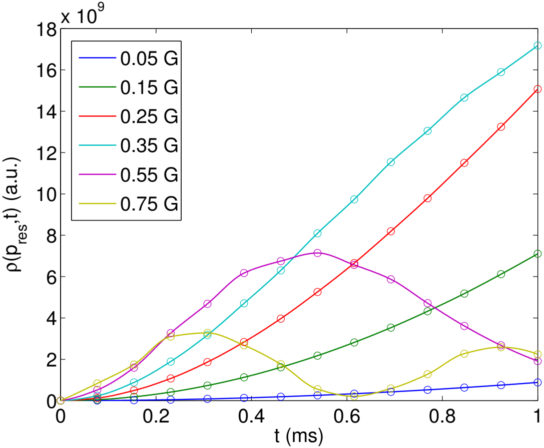
Transition Probability Density: Oscillation Amplitude Dependence



Resonant growth stops above some B_{osc}. The $\rho(p_{\text{res}}, t)$, satisfying the resonance condition, no longer grows resonantly.

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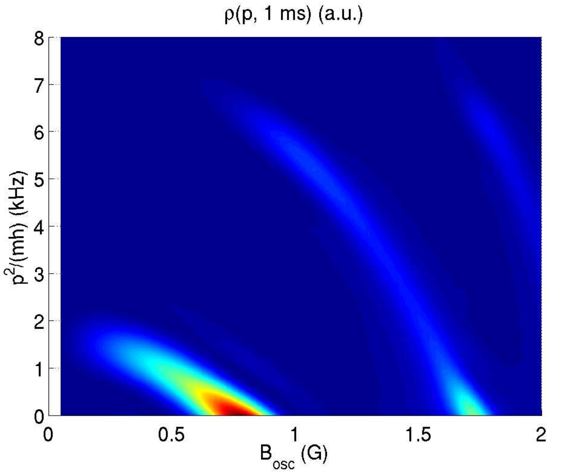

- Negative shift in resonant frequency as B_{osc} increases.
- 'Island' of constructive interference at high B_{osc} gives revival in conversion efficiency.

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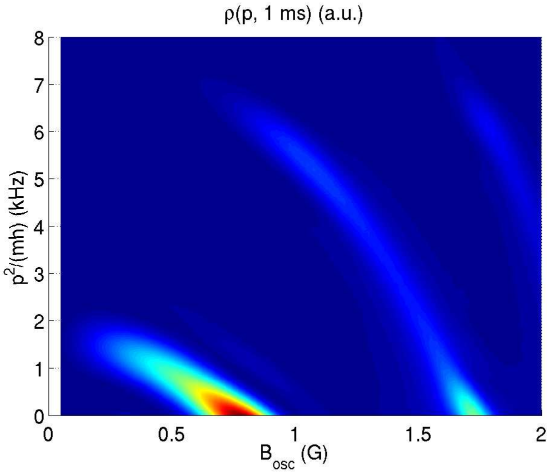

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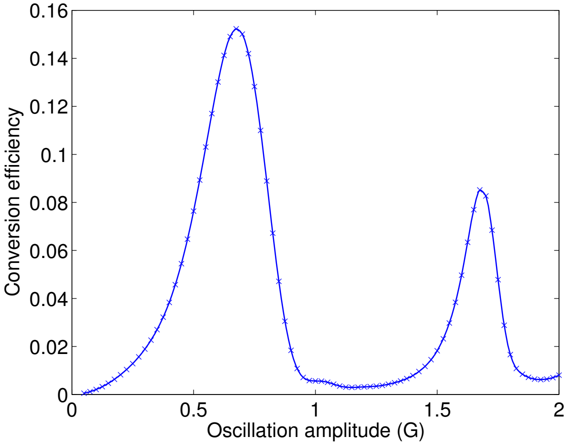

1ms, 6.5 kHz pulses,
$B_{\mathrm{av}} = 156.392G$,
T = 20 nK.

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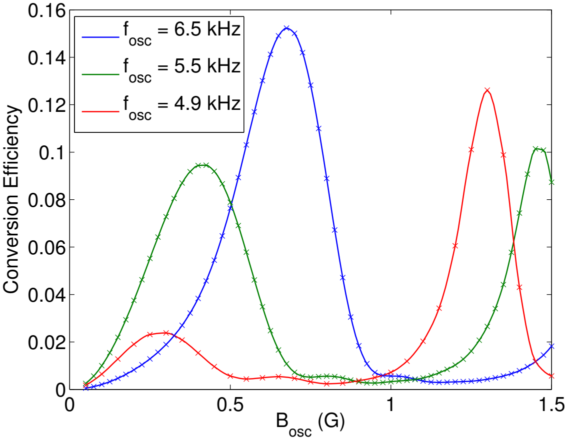

B_{osc} of maxima and minima is

- Strongly dependent on ω_{osc} and B_{av}.
- Only weakly dependent on T.

But absolute conversion is strongly dependent on T.

$\Rightarrow$ Match T, ω_{osc}, and B_{osc}.

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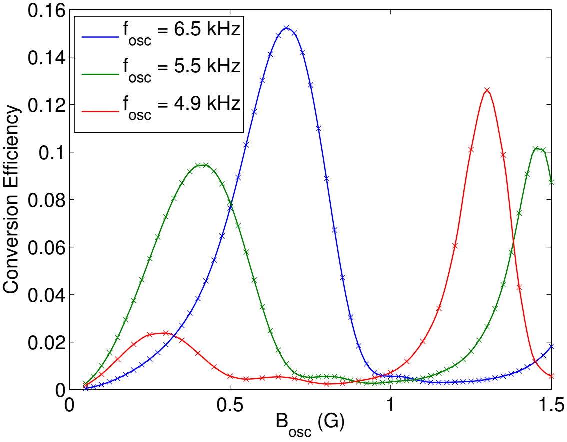

$B_{\rm osc}$ of maxima and minima is

- Strongly dependent on $\omega_{\rm osc}$ and $B_{\rm av}$.
- Only weakly dependent on T.

But absolute conversion is strongly dependent on T.

$\Rightarrow$ Match T, $\omega_{\rm osc}$, and $B_{\rm osc}$.

Conclusions and Future Work

- Two-body simulations of resonant coupling between atom pairs and molecules. Good agreement with experiment.
- Temperature and oscillation amplitude dependent shifts in conversion efficiency.
- Maximise conversion through appropriate choice of temperature and oscillation amplitude and frequency.
- Future work:
    - Analytics
    - Inclusion of many-body effects.
    - Talk to the experimentalists...

Acknowledgements and The Resonant Limerick

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preventing the bound state's appearance.
With the right amplitude,
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