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Adiabatic Mach-Zehnder Interferometry on a Quantized Bose-Josephson Junction

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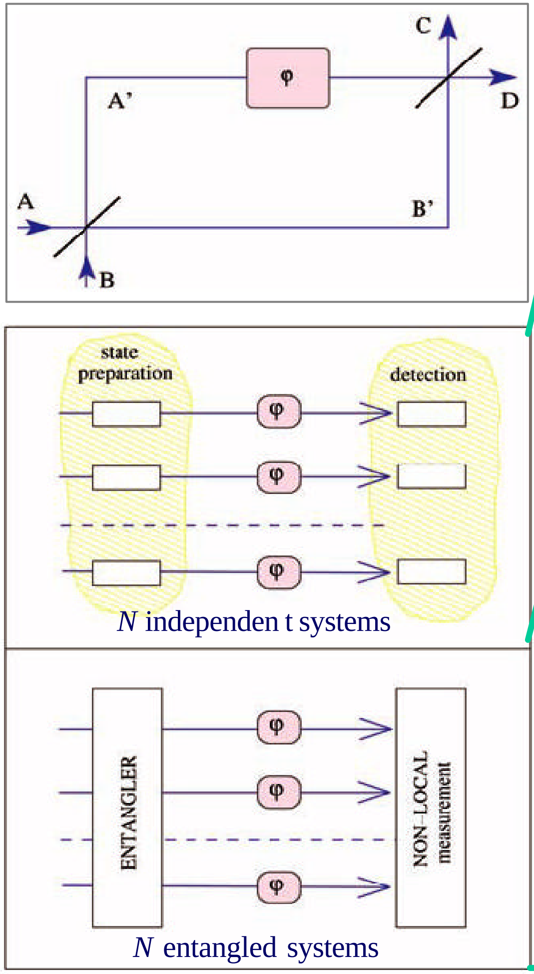
Outline

- Mach-Zehnder interferometer
- Double-well interferometers with BECs
- Nonlinear Kerr effects in condensates
- Many-body effects in systems of ultracold atoms
- Quantized Bose-Josephson junction (QBJJ)
- Mach-Zehnder interferometer on a QBJJ
- Summary and discussions

Mach-Zehnder interferometer



$\Delta j \ \rightarrow$ measurement precision

$$|\psi_{\rm in}\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2}$$

$$|\psi_{\rm out}\rangle = (|0\rangle + e^{i\varphi}|1\rangle)/\sqrt{2}$$

$$p(j) = |\langle\psi_{\rm in}|\psi_{\rm out}\rangle|^2 = \cos^2(\varphi/2)$$

N measurements, $\Delta\varphi = 1/\sqrt{N}$

$$|\phi_{\rm in}\rangle = \frac{1}{\sqrt{2}}(\underbrace{|0\rangle\cdots|0\rangle}_{N \text{ times}} + \underbrace{|1\rangle\cdots|1\rangle}_{N \text{ times}})$$

$$|\phi_{\rm out}\rangle = \frac{1}{\sqrt{2}}(|0\rangle\cdots|0\rangle + e^{iN\varphi}|1\rangle\cdots|1\rangle)$$

$$q(j) = |\langle\psi_{\rm in}|\psi_{\rm out}\rangle|^2 = \cos^2(Nj/2)$$

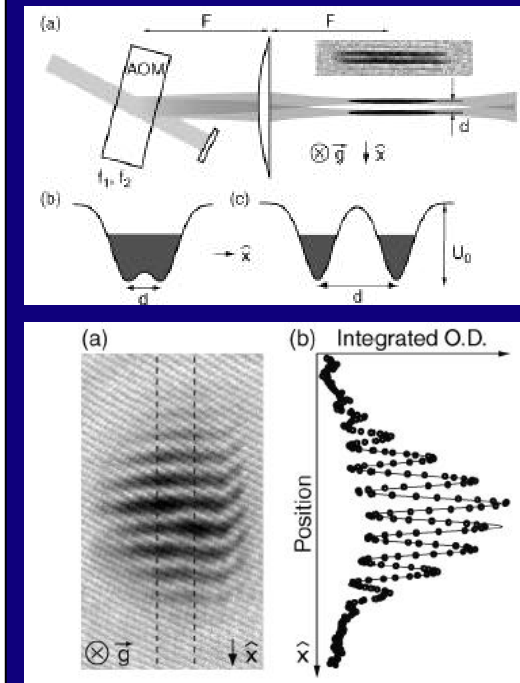
One measurement,

$$\Delta\varphi = \Delta q(\varphi)\Big/\left|\frac{\partial q(\varphi)}{\partial\varphi}\right| = 1/N$$

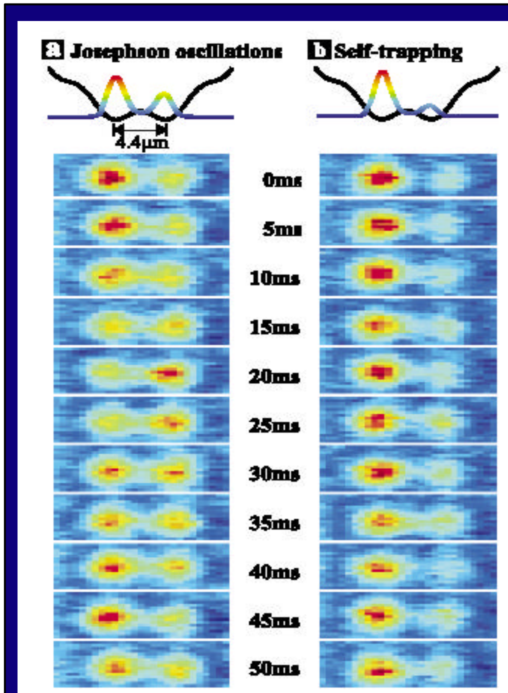
Entanglement enhances the phase measurement precision from the standard quantum limit (or the shot noise limit) to the Heisenberg limit.

V. Giovannetti, S. Lloyd, and L. Maccone, Science 306, 1330 (2004)

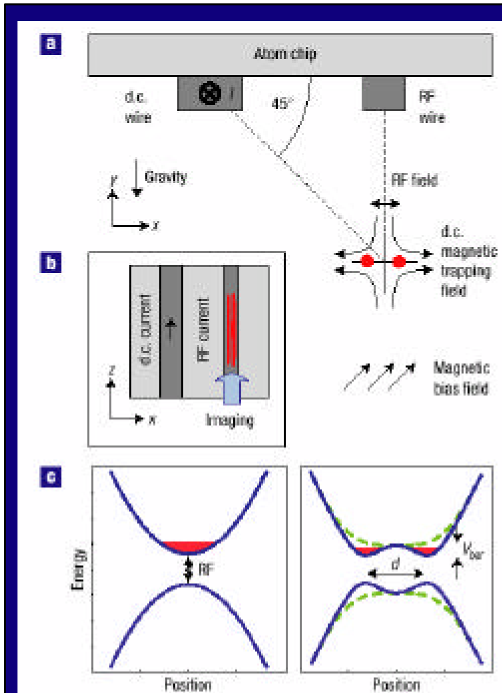
Double-well interferometers with BECs



Ketterle
(MIT)



Oberthaler
(Heidelberg)



Schmiedmayer
(Heidelberg)

Double-well interferometer via combination of magnetic film and bias field, (Hannaford, Sidorov, ACQAO @ SUT)

B.V. Hall *et al*., Phys. Rev. Lett. 98, 030402 (2007)

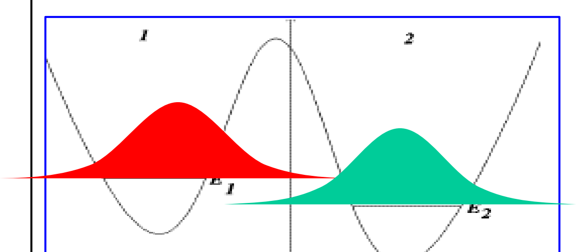
Classical Bose-Josephson junction

Two - mode approximation to the Gross - Pitaevskii equation

$$\Psi(\vec{r},t) = y_1(t)\Phi_1(\vec{r}) + y_2(t)\Phi_2(\vec{r}). \quad y_n(t) = \sqrt{N_n(t)}\; e^{if_n(t)}$$

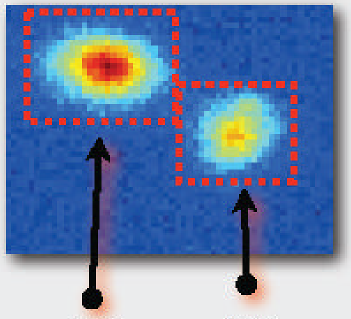
$$i\hbar\frac{\partial}{\partial t} y_1(t) = [E_1 + G_{11}N_1(t) + G_{12}N_2(t)]y_1(t)$$
$$-[K + G_{1,12}N_1(t) + G_{2,12}N_2(t)]y_2(t),$$

$$i\hbar\frac{\partial}{\partial t} y_2(t) = [E_2 + G_{22}N_2(t) + G_{21}N_1(t)]y_2(t)$$
$$-[K + G_{1,12}N_1(t) + G_{2,12}N_2(t)]y_1(t).$$



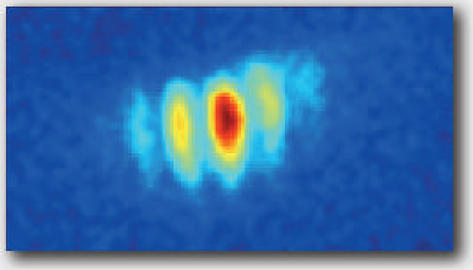
Two types of measurement

- NUMBER:
  large initial separation & short TOF
  (clouds distinct)
  image to count N



$$\Delta N = N_1 - N_2$$

- PHASE:
  small initial separation & long TOF
  (clouds interfere)
  image to measure phase



$$n(x) \sim 1 + a\cos(kx + \phi)$$

Nonlinear Kerr effects in condensates

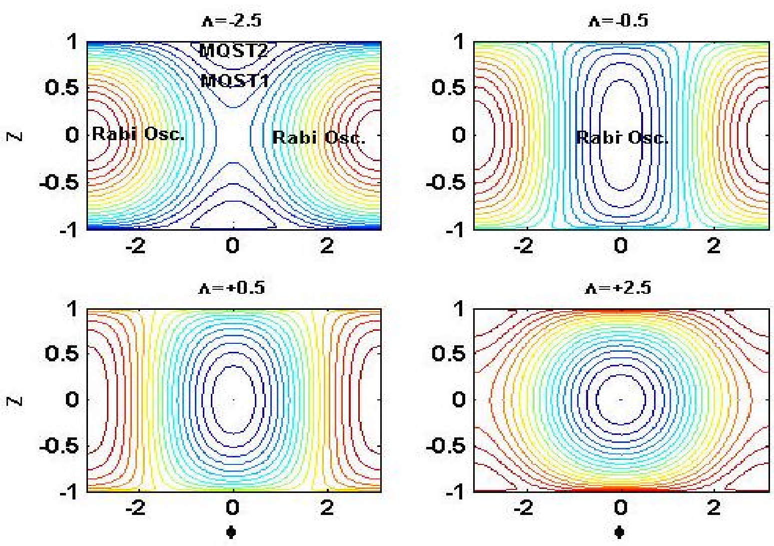
- Inter-atom scattering – s-wave scattering

  - attractive interaction, bright solitons

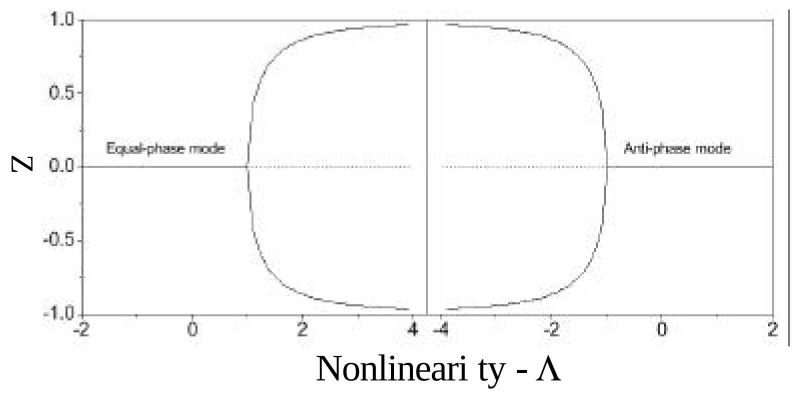
  - repulsive interaction, dark/grey solitons

- Nonlinear effects in classical Bose-Josephson junctions

  - macroscopic quantum self-trapping (MQST) and bistability

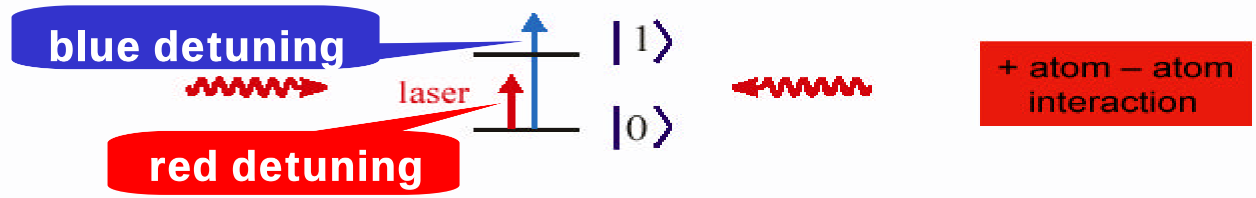


Hopf bifurcation from single stability to bistability



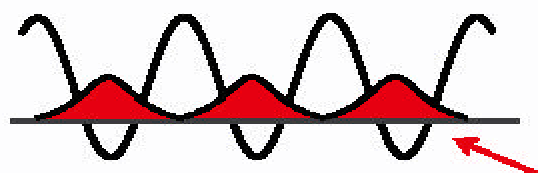
Many-body effects in systems of ultracold atoms

Ultracold Bose atoms in optical lattices – Bose-Hubbard model



$$H = \int d^3x\, \psi^\dagger(\mathbf{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + \underbrace{V_0(\mathbf{x})}_{\text{conservative optical potential}} + V_T(\mathbf{x}) \right) \psi(\mathbf{x}) + \underbrace{\frac{1}{2}\frac{4\pi a_s \hbar^2}{m} \int d^3x\, \psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\psi(\mathbf{x})}_{\text{two particle interaction}}$$

decomposition



tight-binding limit

$$\psi(\vec{x}) = \sum_\alpha w(\vec{x} - \vec{x}_\alpha)\, b_\alpha$$

Wannier functions

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

Jacksch, Zoller, et al., Phys. Rev. Lett. 81, 3108 (1998)

Quantum phase transition

kinetic energy term dominates:

Weakly interacting bosonic gas

-> Superfluidity

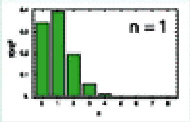
- Atoms are **delocalized** over the entire lattice

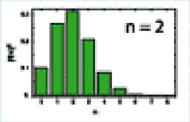
$$|\Psi_{SF}\rangle \propto \left(\sum_{i=1}^{M} \hat{a}_i^{\dagger}\right)^N |0\rangle$$

- Coherence, manybody state can be described by a **macroscopic wavefunction**

$$\langle a_i \rangle \neq 0$$

- **Coherent state** Superposition with a Binomial atom number distribution per lattice site -> number fluctuations





- **Gapless excitation spectrum**

interaction energy term dominates:

Strongly corrolated bosonic system

-> Mott insulator

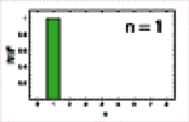
- Atoms are completely **localized** to lattice sites

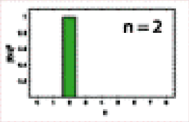
$$|\Psi_{Mott}\rangle \propto \prod_{i=1}^{M} \left(\hat{a}_i^{\dagger}\right)^n |0\rangle$$

- No coherence, no macroscopic wavefunction

$$\langle a_i \rangle = 0$$

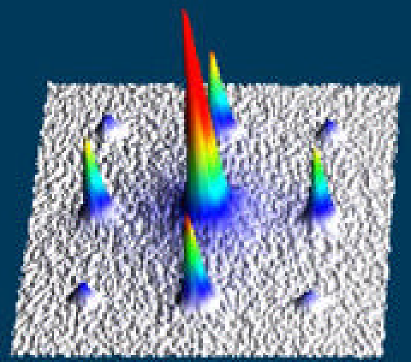
- **Fock state** with a vanishing number fluctuation per lattice site





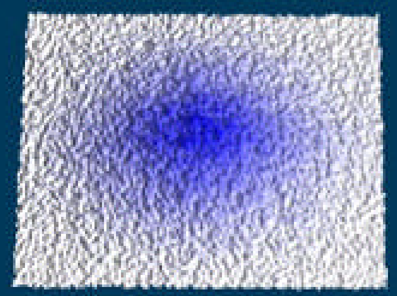
- **Excitation spectrum has an energy gap** $\Delta = U$

Momentum distribution



superfluid

insulator



Bloch, Mainz

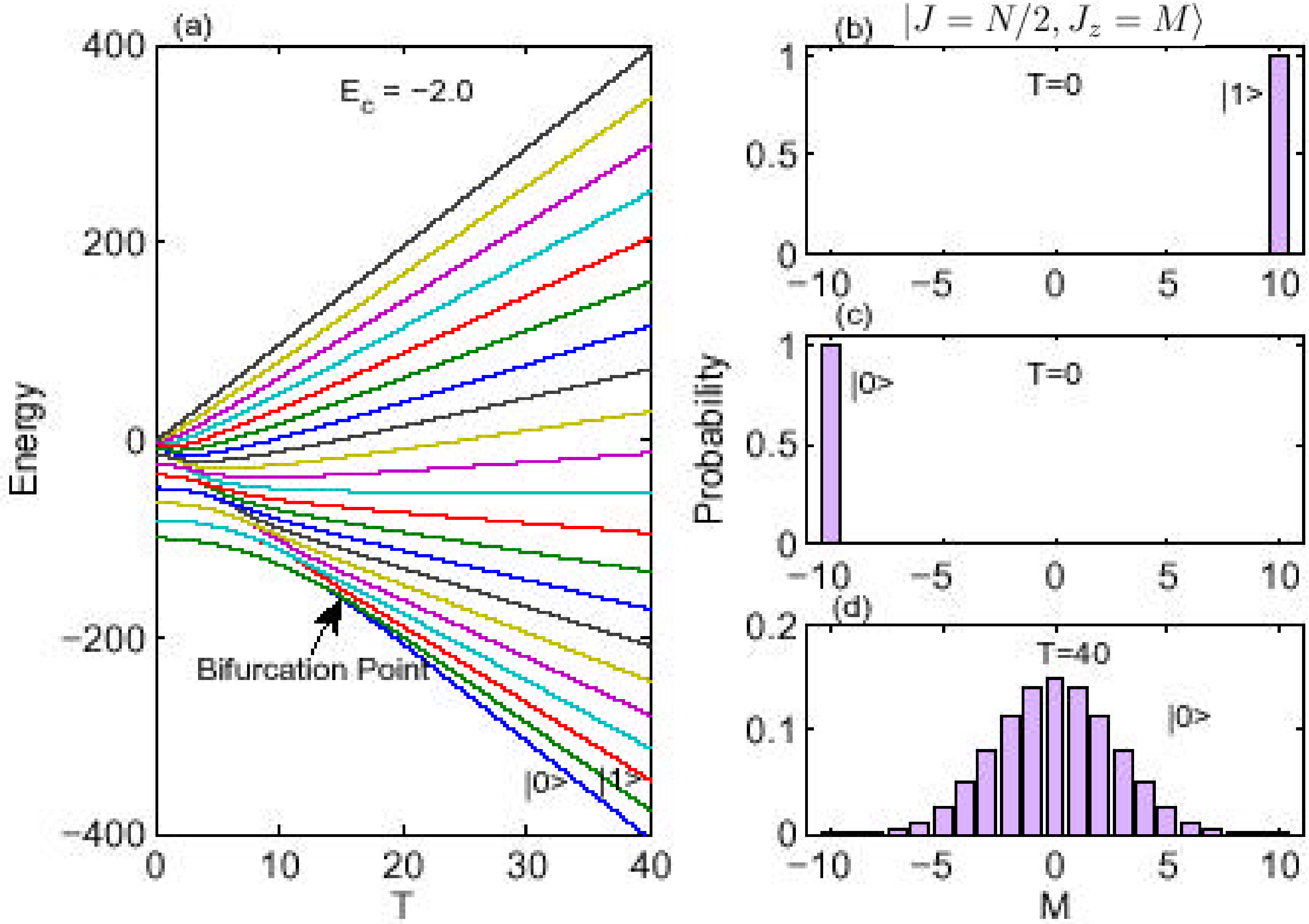
Quantized Bose-Josephson junction

Quantized Hamiltonian for two linearly coupled Bose modes

$$\boxed{H = \frac{d}{2}(n_2 - n_1) - \frac{T}{2}(a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8}(n_2 - n_1)^2}$$

Regime	$\left\lvert E_C / T \right\rvert >> 1$ $E_C > 0$	$\left\lvert E_C / T \right\rvert \approx 0$	$\left\lvert E_C / T \right\rvert >> 1$ $E_C < 0$
State form	$\frac{\left(a_1^+\right)^{N/2}\left(a_2^+\right)^{N/2}\lvert 0\rangle}{(N/2)!}$	$\frac{\left(a_1^+ + a_2^+\right)^N \lvert 0\rangle}{2^{N/2}\sqrt{N!}}$	$\frac{\left(\left(a_1^+\right)^N + \left(a_2^+\right)^N\right)\lvert 0\rangle}{2^{1/2}\sqrt{N!}}$
Coherent matrix $\left\langle a_i^+ a_j \right\rangle$	$\frac{N}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\frac{N}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\frac{N}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Fluctuations	$\Delta N_i \sim 0$	$\Delta N_i \sim \sqrt{N}$	$\Delta N_i \sim N$

Energy spectrum for a symmetric QBJJ of negative charging energy

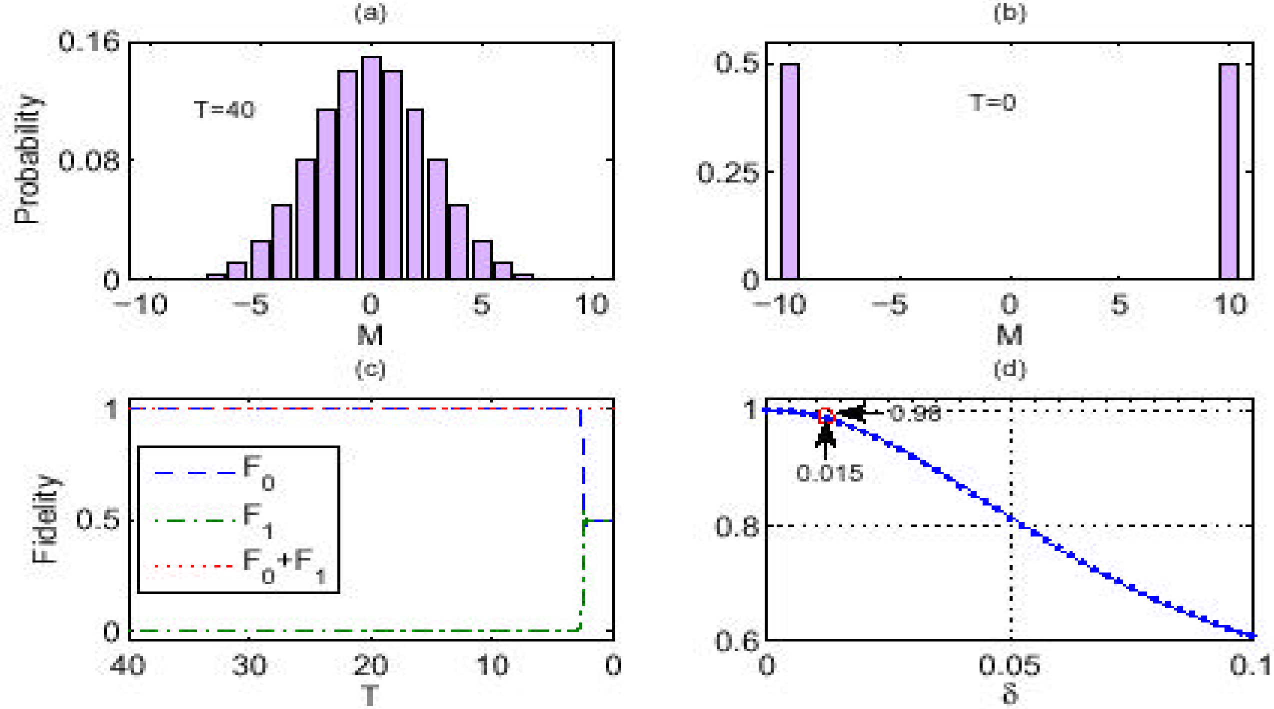


The ground state degeneracy breaks down near the Hopf bifurcation point in the classical Bose-Josephson junction.

C. Lee *et al.*, Phys. Rev. A 69, 033611 (2004) (classical bifurcation)

Mach-Zehnder interferometer on a QBJJ

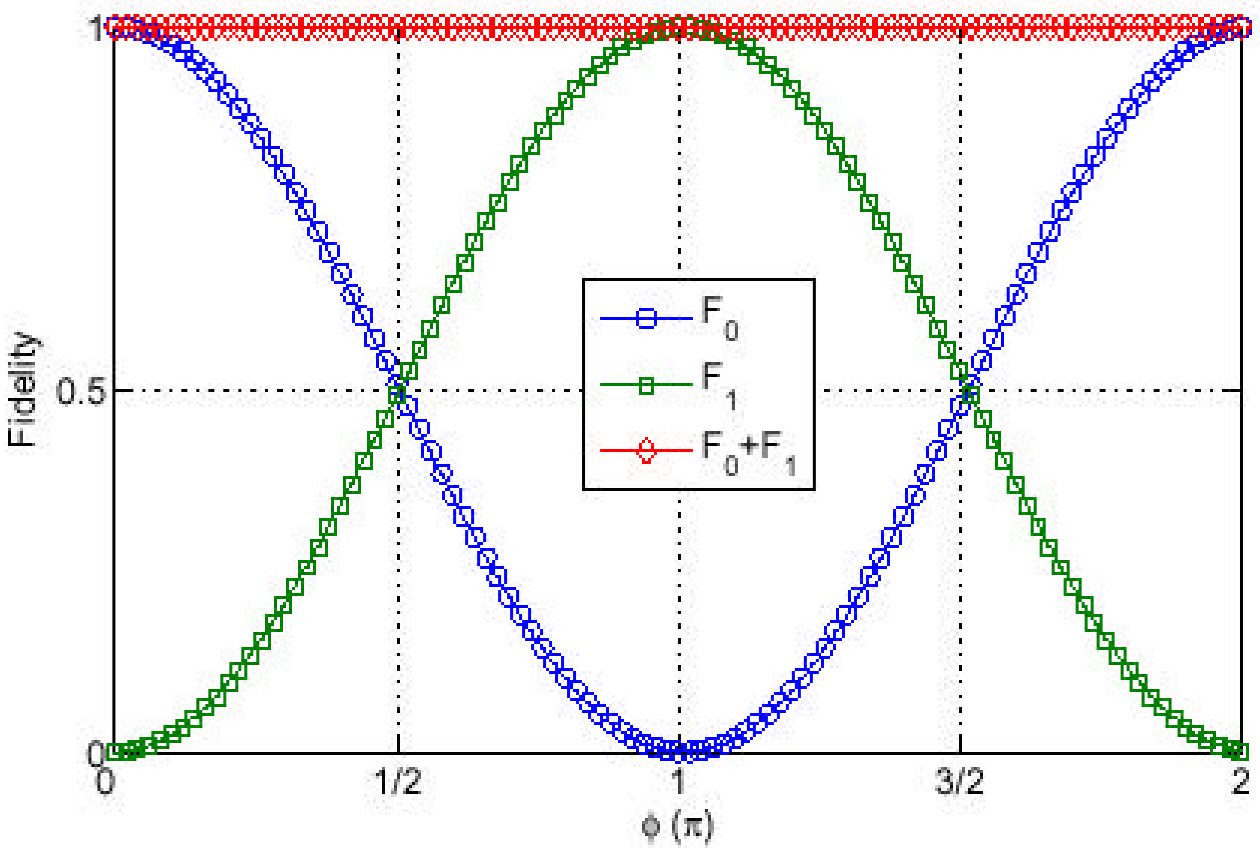
Beam splitting and recombination via dynamical bifurcation



For a symmetric QBJJ with negative charging energy E_c, its ground state |0> is transformed into the equal-probability superposition of the degenerated ground state |0> and the first excited state |1>, which is a path-entangled state (|N/2,-N/2>+|N/2,+N/2>)/√2, when it adiabatically evolves through the bifurcation point from an SU(2) coherent state for the strong tunneling limit.

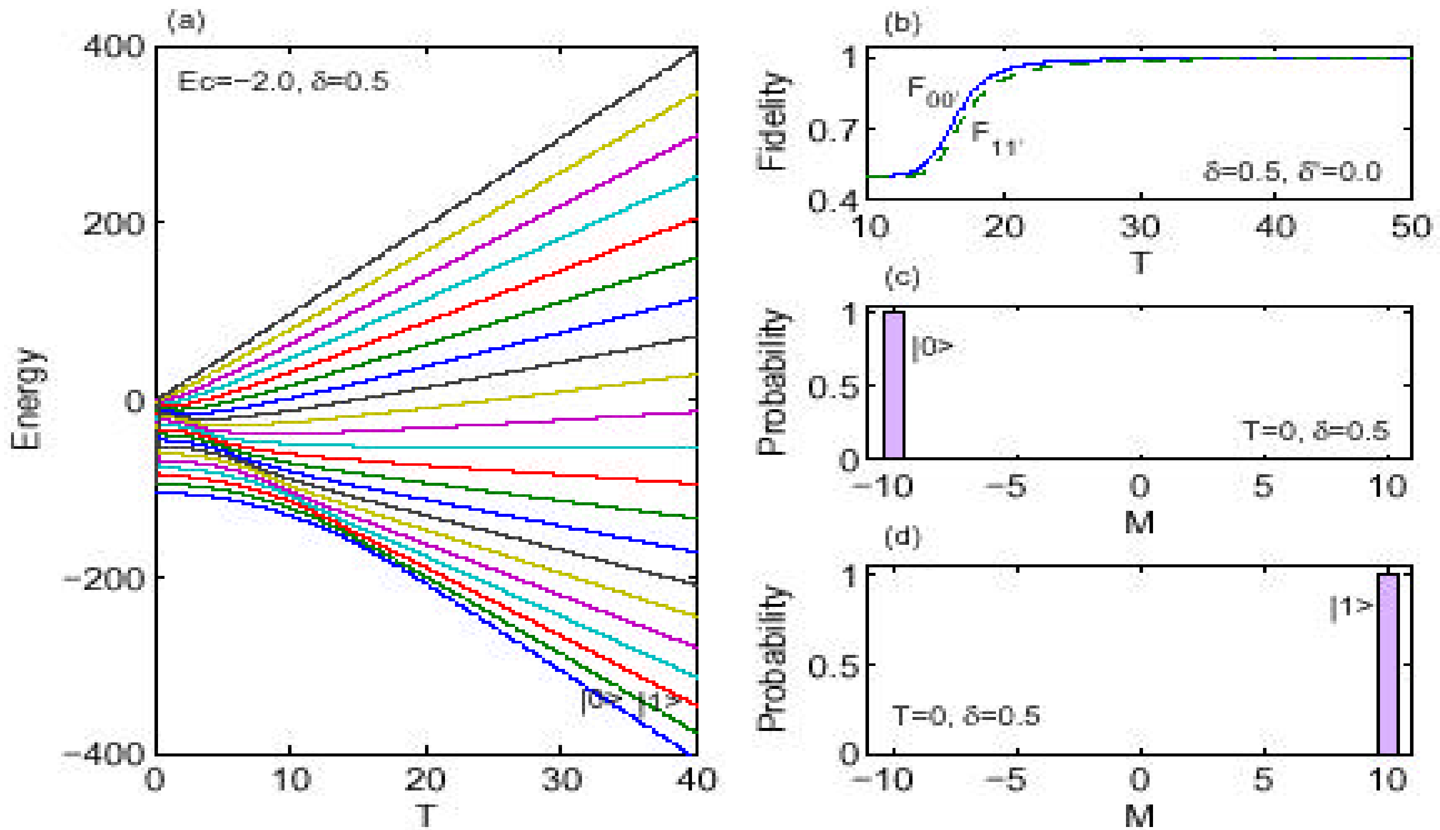
Thus, we can use |0> and |1> as two paths of a MZ interferometer.

Interference via adiabatic recombination



After inducing an unknown phase difference between two paths with a mode-dependent force, the two paths are recombined by adiabatically transforming from the weak coupling limit to the strong coupling limit. The final populations in |0> and |1> show Mach-Zehnder interference behavior determined by the phase shift.

Detection via counting the number of atoms in an asymmetric QBJJ



It is not easy to distinguish |0> and |1> after recombination (the strong coupling limit). However, it is easy to distinguish and detect |N/2,-N/2> and |N/2,+N/2> via atom number counting. Thus we introduce a proper asymmetry after recombination, and then adiabatically decrease the coupling to the weak coupling limit of |0>=|N/2,-N/2> and |1>=|N/2,+N/2>. Due to the absence of energy crossing, the populations in |0> and |1> are unchanged.

Summary and discussions

Summary

- negative charging energy $\rightarrow$ Feshbach resonance
- coupling $\rightarrow$ tunnelling (double-well system), or

  Raman transition (two-component condensate)
- two paths $\rightarrow$ two degenerated ground states for negative charging energy
- beam splitting/recombination $\rightarrow$ dynamical bifurcation
- path entangled state $\rightarrow$ dynamical bifurcation

Discussions

- large total number of particle (in order of 10^3, 10 for systems of photons and trapped ions)
- reduced influence of environment (adiabatic evolution and closed sub-Hilbert space)
- measurement precision of Heisenberg limit (path entangled states)
- experimental possibility (double-well or two-component systems)

C. Lee, Phys. Rev. Lett. 97, 150402 (2006).

Our works related to quantum interference and fluctuations

In processing:

- Phase sensitive excitations
- Quantum and thermal fluctuations

Published:

- Heisenberg limited MZ interference [**Lee**, PRL **97**, 150402 (2006)]
- Discrete vortices melting via quantum fluctuations [**Lee**, Alexander, Kivshar, PRL **97**, 180408 (2006)]
- Bistability and bifurcation [**Lee** et al., PRA **69**, 033611 (2004)]
- Quasispin model for macroscopic quantum tunneling [**Lee** et al., PRA **68**, 053614 (2003)]
- AC Josephson effects [**Lee** et al., PRE **66**, 026202 (2002); PRA **64**, 053604 (2001)]
- Coupled-mode theory [Ostrovskaya, Kivshar, et al., PRA **61**, 031601 (2000)]

Thanks for your attention!