Adiabatic Mach-Zehnder Interferometry on a Quantized Bose-Josephson Junction

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Outline

• Mach-Zehnder interferometer
• Double-well interferometers with BECs
• Nonlinear Kerr effects in condensates
• Many-body effects in systems of ultracold atoms
• Quantized Bose-Josephson junction (QBJJ)
• Mach-Zehnder interferometer on a QBJJ
• Summary and discussions

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Entanglement enhances the phase measurement precision from the standard quantum limit (or the shot noise limit) to the Heisenberg limit.

\[ \Delta \varphi \rightarrow \text{measurement precision} \]

\[ |\psi_{\text{in}}\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \]

\[ |\psi_{\text{out}}\rangle = (|0\rangle + e^{i\varphi}|1\rangle)/\sqrt{2} \]

\[ p(\varphi) = |\langle \psi_{\text{in}} | \psi_{\text{out}} \rangle|^2 = \cos^2(\varphi/2) \]

One measurement,

\[ \Delta \varphi = \Delta q(\varphi) / \left| \frac{\partial q(\varphi)}{\partial \varphi} \right| = 1/N \]

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Double-well interferometers with BECs

Ketterle (MIT)  Oberthaler (Heidelberg)  Schmiedmayer (Heidelberg)

Double-well interferometer via combination of magnetic film and bias field, (Hannaford, Sidorov, ACQAO @ SUT)

Classical Bose-Josephson junction

Two-mode approximation to the Gross-Pitaevskii equation

$$\Psi(\vec{r}, t) = \psi_1(t) \Phi_1(\vec{r}) + \psi_2(t) \Phi_2(\vec{r}). \quad \psi_n(t) = \sqrt{N_n(t)} \ e^{i\phi_n(t)}$$

$$i\hbar \frac{\partial}{\partial t} \Psi_1(t) = [E_1 + G_{11} N_1(t) + G_{12} N_2(t)]\psi_1(t)$$

$$- [K + G_{1,12} N_1(t) + G_{2,12} N_2(t)]\psi_2(t),$$

$$i\hbar \frac{\partial}{\partial t} \Psi_2(t) = [E_2 + G_{22} N_2(t) + G_{21} N_1(t)]\psi_2(t)$$

$$- [K + G_{1,12} N_1(t) + G_{2,12} N_2(t)]\psi_1(t).$$
Two types of measurement

- **NUMBER:**
  large initial separation & short TOF (clouds distinct)
  image to count $N$

- **PHASE:**
  small initial separation & long TOF (clouds interfere)
  image to measure phase

$$\Delta N = N_1 - N_2$$

$$n(x) \sim 1 + a \cos (kx + \phi)$$
Nonlinear Kerr effects in condensates

- Inter-atom scattering – s-wave scattering
  - attractive interaction, bright solitons
  - repulsive interaction, dark/grey solitons
- Nonlinear effects in classical Bose-Josephson junctions
  - macroscopic quantum self-trapping (MQST) and bistability

Hopf bifurcation from single stability to bistability
Many-body effects in systems of ultracold atoms

Ultracold Bose atoms in optical lattices – Bose-Hubbard model

\[ H = \int d^3 x \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{o}(x) + V_{T}(x) \right) \psi(x) + \frac{1}{2} \frac{4\pi a_s \hbar^2}{m} \int d^3 x \, \psi^\dagger(x) \psi^\dagger(x) \psi(x) \psi(x) \]

conservative optical potential

two particle interaction

decomposition

\[ \psi(\vec{x}) = \sum_{\alpha} \omega(\vec{x} - \vec{x}_\alpha) b_\alpha \]

tight-binding limit

\[ H = -J \sum_{\langle i, j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1) \]

Quantum phase transition

**kinetic energy term dominates:**

Weakly interacting bosonic gas

\[ |\Psi_{SF}\rangle \propto \left( \sum_{i=1}^{N} a_i^+ \right)^N |0\rangle \]

- Atoms are **delocalized** over the entire lattice
- Coherence, manybody state can be described by a **macroscopic wavefunction**
  \[ \langle a_i \rangle \neq 0 \]
- Coherent state
  Superposition with a Binomial atom number distribution per lattice site
  \( \rightarrow \) number fluctuations

- Gapless excitation spectrum

**interaction energy term dominates:**

Strongly correlated bosonic system

\[ |\Psi_{Mott}\rangle \propto \prod_{j=1}^{M} (a_j^\dagger)^n |0\rangle \]

- Atoms are completely **localized** to lattice sites
- No coherence, no macroscopic wavefunction
  \[ \langle a_i \rangle = 0 \]
- Fock state
  with a vanishing number fluctuation per lattice site

- Excitation spectrum has an energy gap \( \Delta = U \)

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**Momentum distribution**

Superfluid

\[ \rightarrow \] Mott insulator

**Bloch, Mainz**
Quantized Bose-Josephson junction

Quantized Hamiltonian for two linearly coupled Bose modes

\[ H = \frac{\delta}{2} (n_2 - n_1) - \frac{T}{2} (a_2^+ a_1 + a_1^+ a_2) + \frac{E_C}{8} (n_2 - n_1)^2 \]

| Regime               | \(|E_C / T| \gg 1\) | \(|E_C / T| \approx 0\) | \(|E_C / T| \gg 1\) |
|----------------------|---------------------|---------------------|---------------------|
|                      | \(E_C > 0\)         | \(E_C < 0\)         | \(E_C < 0\)         |
| State form           | \(\frac{(a_1^+)^N (a_2^+)^{N/2}}{(N/2)!} | 0\) \) | \(\frac{(a_1^+ + a_2^+)^N}{2^{N/2} \sqrt{N}!} | 0\) \) | \(\frac{(a_1^+)^N + (a_2^+)^N}{2^{1/2} \sqrt{N}!} | 0\) \) |
| Coherent matrix      | \(N \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \) | \(N \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \) | \(N \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix} \) |
| \(\langle a_i^+ a_j \rangle\) | \(\Delta N_i \sim 0\) | \(\Delta N_i \sim \sqrt{N}\) | \(\Delta N_i \sim N\) |

Fluctuations
Energy spectrum for a symmetric QBJJ of negative charging energy

The ground state degeneracy breaks down near the Hopf bifurcation point in the classical Bose-Josephson junction.

For a symmetric QBJJ with negative charging energy $E_c$, its ground state $|0\rangle$ is transformed into the equal-probability superposition of the degenerated ground state $|0\rangle$ and the first excited state $|1\rangle$, which is a path-entangled state $(|N/2,-N/2\rangle + |N/2,+N/2\rangle)/\sqrt{2}$, when it adiabatically evolves through the bifurcation point from an SU(2) coherent state for the strong tunneling limit.

Thus, we can use $|0\rangle$ and $|1\rangle$ as two paths of a MZ interferometer.
After inducing an unknown phase difference between two paths with a mode-dependent force, the two paths are recombined by adiabatically transforming from the weak coupling limit to the strong coupling limit. The final populations in $|0\rangle$ and $|1\rangle$ show Mach-Zehnder interference behavior determined by the phase shift.
Detection via counting the number of atoms in an asymmetric QBJJ

It is not easy to distinguish $|0\rangle$ and $|1\rangle$ after recombination (the strong coupling limit). However, it is easy to distinguish and detect $|N/2,-N/2\rangle$ and $|N/2,+N/2\rangle$ via atom number counting. Thus we introduce a proper asymmetry after recombination, and then adiabatically decrease the coupling to the weak coupling limit of $|0\rangle=|N/2,-N/2\rangle$ and $|1\rangle=|N/2,+N/2\rangle$. Due to the absence of energy crossing, the populations in $|0\rangle$ and $|1\rangle$ are unchanged.
## Summary and discussions

### Summary
- negative charging energy $\rightarrow$ Feshbach resonance
- coupling $\rightarrow$ tunnelling (double-well system), or
  - Raman transition (two-component condensate)
- two paths $\rightarrow$ two degenerated ground states for negative charging energy
- beam splitting/recombination $\rightarrow$ dynamical bifurcation
- path entangled state $\rightarrow$ dynamical bifurcation

### Discussions
- large total number of particle (in order of $10^3$, 10 for systems of photons and trapped ions)
- reduced influence of environment (adiabatic evolution and closed sub-Hilbert space)
- measurement precision of Heisenberg limit (path entangled states)
- experimental possibility (double-well or two-component systems)

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Our works related to quantum interference and fluctuations

In processing:
- Phase sensitive excitations
- Quantum and thermal fluctuations

Published:
- Heisenberg limited MZ interference [Lee, PRL 97, 150402 (2006)]
- Discrete vortices melting via quantum fluctuations [Lee, Alexander, Kivshar, PRL 97, 180408 (2006)]
- Bistability and bifurcation [Lee et al., PRA 69, 033611 (2004)]
- Quasispin model for macroscopic quantum tunneling [Lee et al., PRA 68, 053614 (2003)]
- AC Josephson effects [Lee et al., PRE 66, 026202 (2002); PRA 64, 053604 (2001)]
- Coupled-mode theory [Ostrovskaya, Kivshar, et al., PRA 61, 031601 (2000)]

Thanks for your attention!