Criteria for Bohm's version of the EPR paradox
- *EPR paradox for spins*

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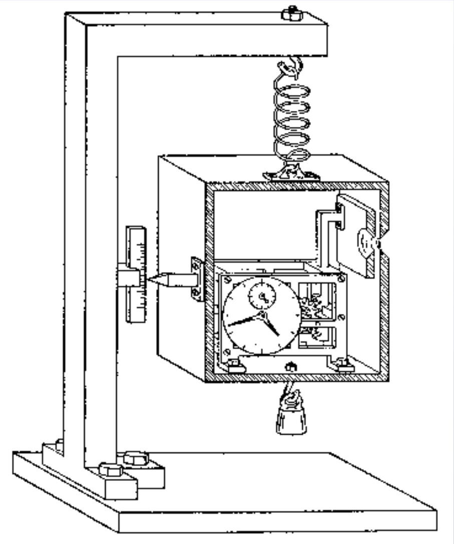
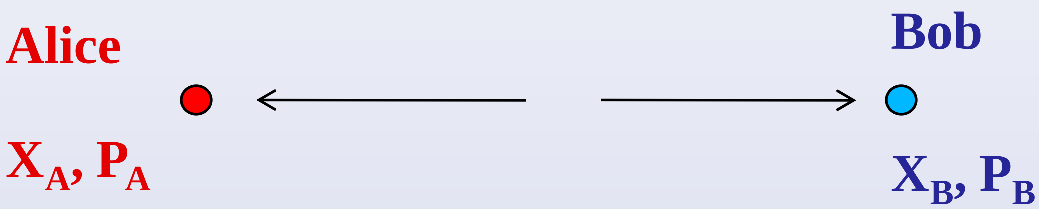
The Einstein-Bohr debates



The Einstein-Podolsky-Rosen paradox



- Necessary condition for **Completeness**:

  "*Every element of the physical reality must have a counterpart in the physical theory*".

- EPR's sufficient condition for **Reality** :

  Accurate prediction of a physical quantity at a distance → element of reality associated to it.

- **Local Causality**:

  No action at a distance

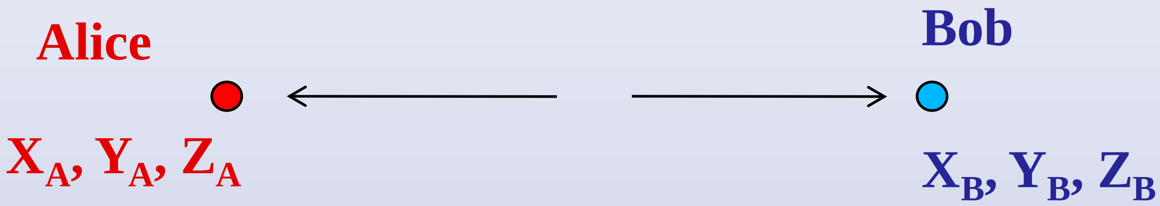
- **Quantum Mechanics** predicts, for certain entangled states, $X_A = X_B$ and $P_A = - P_B$; by measuring at A one can predict with certainty either X_B or P_B .

- EPR conclude that ***Quantum Mechanics is incomplete***.

Bohr’s reply

“Blah blah blah mambo jambo mambo blah blah... *Uncontrolable interaction...* Mambo jambo blah blah... *Complementarity...* Yada yada yada...”

Bohm’s version of the EPR paradox



Essentially the same, but with spin-1/2 particles in a singlet state:

$X_A = -X_B,\ Y_A = -\ Y_B,\ Z_A = -\ Z_B$

But neither the correlations in EPR or in Bohm’s argument are experimentally feasible…

EPR-Reid criterion (1989)

Extension of EPR's sufficient condition for Reality:

Imperfect prediction of a physical quantity at a distance → probabilistic element of reality associated to it.

Alice measures X_A: $P(x_B|x_A)$, $\Delta^2(x_B|x_A)$

P_A: $P(p_B|p_A)$, $\Delta^2(p_B|p_A)$

Alice
X_A, P_A

Bob
X_B, P_B

Conditional inference variances

$$\Delta^2_{inf}(x_B|X_A) = \sum_{x_A} P(x_A)\Delta^2(x_B|x_A)$$

$$\Delta^2_{inf}(p_B|P_A) = \sum_{p_A} P(p_A)\Delta^2(p_B|p_A)$$

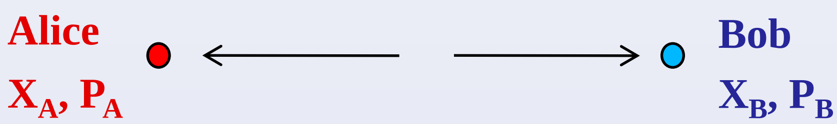
HUP:

$\Delta^2 x\, \Delta^2 p \geq 1$

$$\Delta^2_{inf}(x_B|X_A)\Delta^2_{inf}(p_B|P_A) \geq 1$$

Violation implies **local causality** or **completeness** (or both) are false

More formally...



Local Causality (Bell)

$$P(x_A, x_B | X_A, X_B, \lambda) = P(x_A | X_A, \lambda) P(x_B | X_B, \lambda)$$

where λ specifies any relevant variables that locally (probabilistically) determine the outcomes

Completeness for Bob implies his probabilities are compatible with a quantum state

$$P(x_A, x_B | X_A, X_B, \lambda) = P(x_A | X_A, \lambda) P(x_B | X_B, \underline{\rho_\lambda})$$
$$P(p_A, p_B | P_A, P_B, \lambda) = P(p_A | P_A, \lambda) P(p_B | P_B, \underline{\rho_\lambda})$$

If one can predict x_B or p_B with certainty $P(x_B | X_B, \rho_\lambda) \in \{0, 1\}$

$$P(p_B | P_B, \rho_\lambda) \in \{0, 1\}$$

But no quantum state simultaneously allows these probabilities $\Rightarrow$ ***inconsistency!***

Steering and the EPR-Reid criterion

[Wiseman et. al., Phys. Rev. Lett. 98, 140402 (2007)]

Alice CANNOT *STEER* Bob's state iff there's a Local Hidden State (LHS) model for Bob:

$$P(a,b|A,B) = \sum_\lambda P(\lambda)P(a|A,\lambda)P(b|B,\rho_\lambda)$$

for all outcomes *a, b* of all measurements *A, B* that Alice and Bob can respectively make

But this is essentially the **encapsulation of Local Causality and Completeness**, allowing for the most general distribution of "elements of reality" as is compatible with those assumptions.

That alone (without any further criteria for Reality as in EPR) leads to a **contradiction with Quantum Mechanics**, even in the case of imperfect correlations.

Steering and the EPR-Reid criterion

Alice
X_A, P_A

Bob
X_B, P_B

$$\boxed{P(x_A, x_B|X_A, X_B) = \sum_\lambda P(\lambda)P(x_A|X_A,\lambda)P(x_B|X_B,\rho_\lambda)}$$

From this we can show that

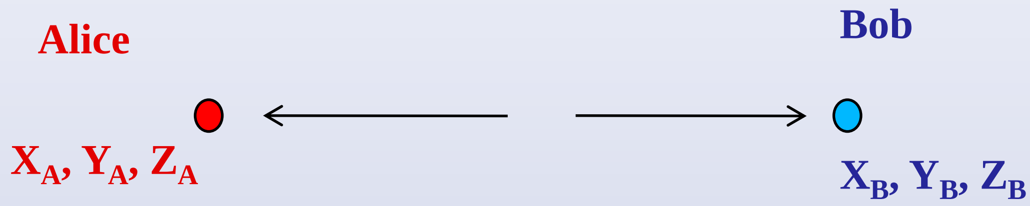
$$\Delta^2_{inf}(x_B|X_A) \geq \sum_\lambda P(\lambda)\Delta^2(x_B|\rho_\lambda)$$

Now applying the Cauchy-Schwarz inequality and H.U.P.

$$\Delta^2_{inf}(x_B|X_A)\Delta^2_{inf}(p_B|P_A) \geq \left\{\sum_\lambda P(\lambda)\Delta(x_B|\rho_\lambda)\Delta(p_B|\rho_\lambda)\right\}^2 \geq 1$$

That's the EPR-Reid criterion

Criteria for EPR-Bohm paradox



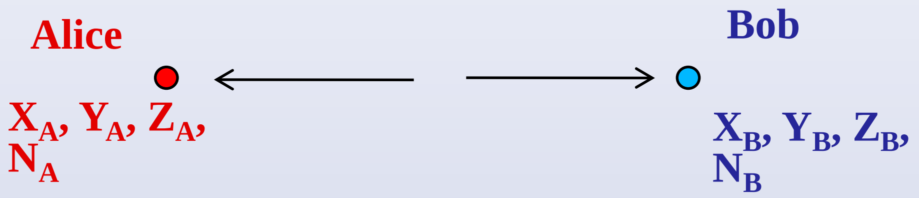
$$\boxed{P(x_A, x_B | X_A, X_B) = \sum_\lambda P(\lambda) P(x_A | X_A, \lambda) P(x_B | X_B, \rho_\lambda)}$$

H.U.P. now is:

$$\Delta(x|\rho_\lambda)\Delta(y|\rho_\lambda) \geq \frac{1}{2}\left|\langle z \rangle_{\rho_\lambda}\right|$$

$$\boxed{\Delta_{inf}(x_B|X_A)\Delta_{inf}(y_B|Y_A) \geq \sum_{z_A} \frac{P(z_A)}{2} \left|\langle z_B | z_A \rangle\right|}$$

Criteria for EPR-Bohm paradox



Using bosonic Schwinger spins

$$X_A = (a_- a_+^\dagger + a_-^\dagger a_+)/2$$

$$Y_A = (a_- a_+^\dagger - a_-^\dagger a_+)/2i$$

$$Z_A = (a_+^\dagger a_+ - a_-^\dagger a_-)/2$$

$$N_A = (a_+^\dagger a_+ + a_-^\dagger a_-)$$

With the H.U.P.:

$$\Delta^2 x + \Delta^2 y + \Delta^2 z \geq \frac{\Delta^2 N}{4}$$

$$\boxed{\Delta^2_{inf}(x_B|X_A) + \Delta^2_{inf}(y_B|Y_A) + \Delta^2_{inf}(z_B|Z_A) \geq \frac{\langle N_B \rangle}{2}}$$

Applications (Bohm's example)

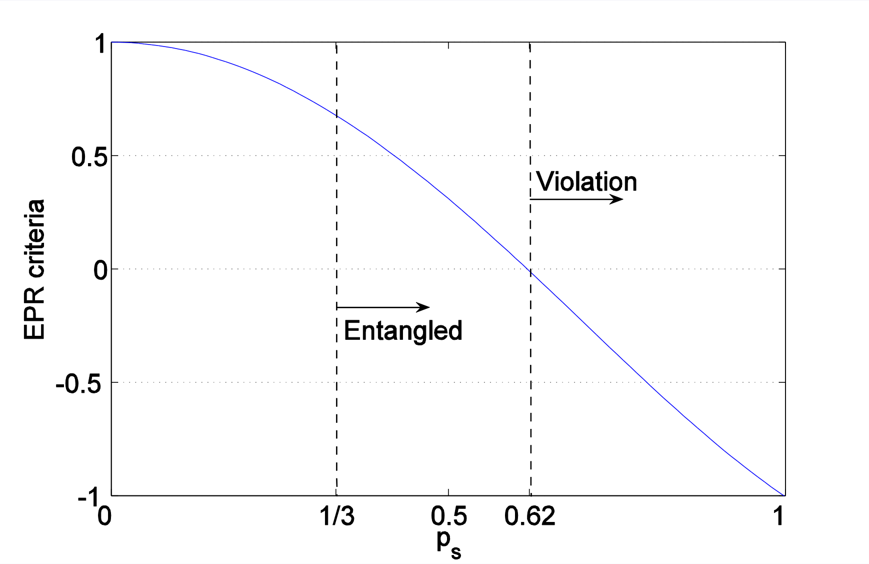
$$\boxed{\Delta_{inf}(x_B|X_A)\Delta_{inf}(y_B|Y_A) \geq \sum_{z_A} \frac{P(z_A)}{2} |\langle z_B|z_A\rangle|}$$

Werner state $\quad \rho_w = (1-p_s)\frac{I}{4} + p_s|\psi_s\rangle\langle\psi_s|$

$$\Delta^2_{inf}(x_B|X_A) = \Delta^2_{inf}(y_B|Y_A) = (1-{p_S}^2)/4$$

$$\sum_{z_A} \frac{P(z_A)}{2} |\langle z_B|z_A\rangle| = p_S/2$$

$$p_S = (\sqrt{5}-1)/2$$



Applications

$$\boxed{\Delta^2_{inf}(x_B|X_A) + \Delta^2_{inf}(y_B|Y_A) + \Delta^2_{inf}(z_B|Z_A) \geq \frac{\langle N_B \rangle}{2}}$$

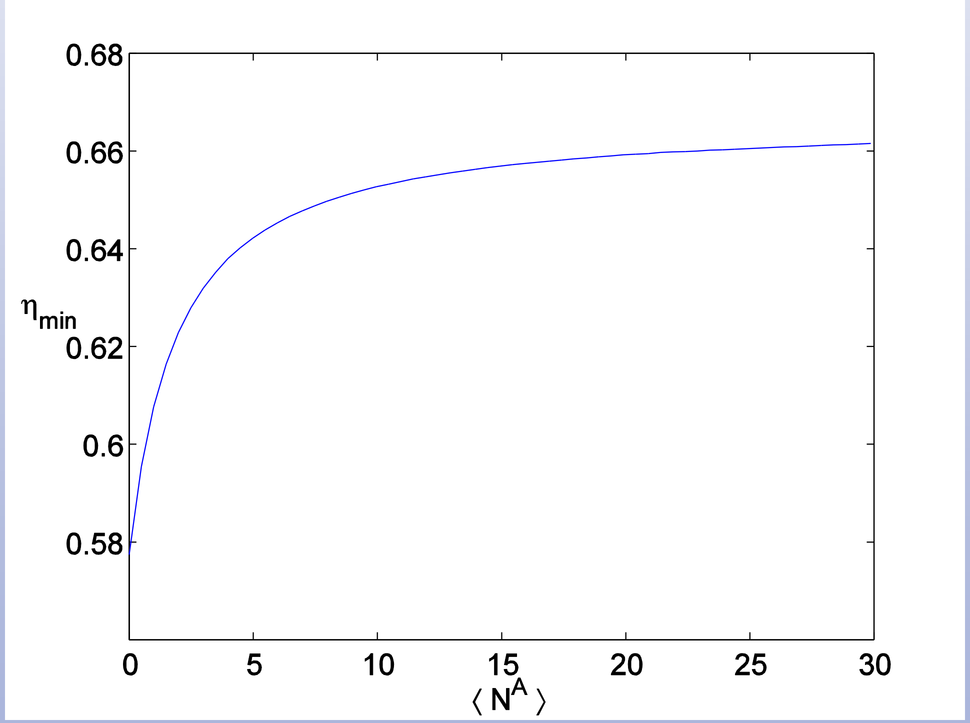
Two parametric amplifiers: $H = i\hbar\kappa(a_+^\dagger b_-^\dagger - a_-^\dagger b_+^\dagger) + H.C.$

$$X_A = (a_- a_+^\dagger + a_-^\dagger a_+)/2$$

$$Y_A = (a_- a_+^\dagger - a_-^\dagger a_+)/2i$$

$$Z_A = (a_+^\dagger a_+ - a_-^\dagger a_-)/2$$

$$N_A = (a_+^\dagger a_+ + a_-^\dagger a_-)$$



Minimum detection efficiency required for violation of criteria as a function of N_B

Three classes of (non-)locality

Local causality
(Bell non-locality)

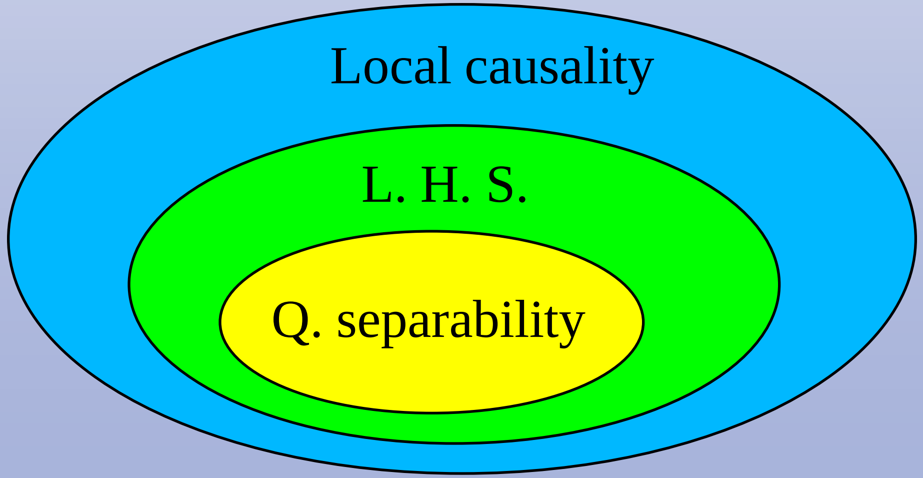
$$P(x_A, x_B|X_A, X_B) = \sum_\lambda P(\lambda)P(x_A|X_A, \underline{\lambda})P(x_B|X_B, \underline{\lambda})$$

Local Hidden State
(EPR paradox)

$$P(x_A, x_B|X_A, X_B) = \sum_\lambda P(\lambda)P(x_A|X_A, \underline{\lambda})P(x_B|X_B, \underline{\rho_\lambda})$$

Quantum separability
(Entanglement)

$$P(x_A, x_B|X_A, X_B) = \sum_\lambda P(\lambda)P(x_A|X_A, \underline{\sigma_\lambda})P(x_B|X_B, \underline{\rho_\lambda})$$



Conclusion

- We derived criteria for the EPR-Bohm paradox (with spins);
- These were argued to follow from the EPR premises of Local Causality and Completeness without the need for EPR's condition for Reality;
- The formalization of such premises is argued to be that of a Local Hidden State model introduced by Wiseman et al;
- The criteria are sufficient for the demonstration of the EPR paradox or analogously, sufficient for demonstration of steering (reduction of the wave packet);
- Open questions:
  - Can better inequalities be derived with this approach?
  - What is the full set of criteria that characterizes a Local Hidden State model?

Thank you!