Entanglement between a laser source and driven qubit

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HIGHER ORDER CORRECTIONS TO THE DICKE SUPERRADIANT PHASE TRANSITION

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The phase transition in the Dicke model for superradiance obtained by Hepp and Lieb and Wang and Hioe is modified by eliminating the rotating wave approximation.

The problem of N two level atoms interacting with a radiation field has been the subject of considerable research. The starting point for many investigators has been the Dicke [1] model which considers the interaction of N two level atoms with a single mode of the radiation field in a cavity of volume V. A variety of solutions to this problem may be found in the literature [2]. Recently an interesting result on this system has been found by Hepp and Lieb [3], who show that in the thermodynamic limit of $N \to \infty$, $V \to \infty$ and for a sufficiently large value of the coupling constant between the atom and field, the system exhibits a second-order phase transition from normal to superradiance at a certain critical temperature. The same result was later obtained in a less rigorous but more transparent manner by Wang and Hioe (W.H.) [4]. Both the analyses men-

and λ' is the coupling between the atoms and the field; a, a^+ are the boson annihilation and creation operators for the field modes and σ_j^+, σ_j^- and σ_j^z are the pseudo spin operators for the jth atom. In the full interaction Hamiltonian we consider $\mu = 1$. The interaction Hamiltonian in the R.W.A. is obtained from eq. (3) by setting $\mu = 0$.

The thermodynamic properties of the above model may be calculated from the canonical partition function $Z(N, T)$ defined by

$$Z(N, T) = \mathrm{Tr}\,\exp(-\beta H); \quad \beta = 1/kT. \tag{4}$$

Following W.H. we use the coherent states $|\alpha\rangle$ [6] as a basis to evaluate the trace over the field variables. This yields

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Driving a Quantum System with the Output Field From Another Driven Quantum System

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Quantum Langevin equations and a master equation are derived for a two-atom system in which the first atom is driven by coherent field, and the fluorescent light used to drive a second atom. We show that the light beams from both atoms are antibunched, and that they are mutually anticorre-

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Quantum Trajectory Theory for Cascaded Open Systems

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The quantum trajectory theory of an open quantum system driven by a photoemissive source is formulated. The formalism is illustrated by applying it to photon scattering from an atom driven by strongly focused coherent light.

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Research on the generation of nonclassical light has been carried out with considerable success for several years. There has been little work, however, on using nonclassical light to excite a second quantum system. Most experiments are concerned either with directly measuring the nonclassical characteristics of a source or using these

parts. My description is made in te wave function for the composite syster the broken time symmetry I allow the A and B to be mediated by a reser Born-Markoff approximation. Figure ple version of the source and driven

Quantum Trajectory Theory for Cascaded Open Systems, H. J. CARMICHAEL, University of Oregon — The theory of open systems is used in quantum optics in two distinct ways: to model sources of light such as lasers and parametric oscillators, and to describe the response of irradiated cavities and atoms. Sometimes the two applications are used in combination; first, statistical properties of the field radiated by a source are calculated, and these are then used in a separate calculation to determine the response of a system that is irradiated by the source. But the usefulness of this approach is quite limited. It is essentially limited to coherent sources and certain broadband sources such as broadband squeezed or chaotic light. There exists no general theory for cascaded open systems – a theory that gives the response of system B to radiation emitted by system A when there exists an open-systems treatment for A and B separately. In this paper I develop such a theory using the quantum trajectory formulation of open systems. The source A and irradiated system B are described by a single stochastic wavefunction. Generally the wavefunction describes an entangled state of the A and B subsystems. In the quantum trajectory formalism the wavefunction undergoes a coherent evolution governed by a nonunitary Schrödinger equation, interrrupted at random times by wavefunction collapses. I derive the nonunitary Schrödinger equation and the form of the collapses. I illustrate the theory with some simple examples and discuss potential applications to problems involving the interaction of atoms with nonclassical light.

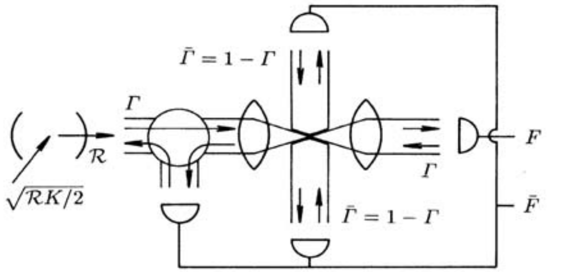


FIG. 2. Input-output channels for an atom driven by coherent light.

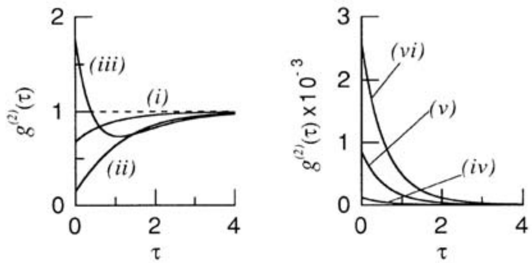


FIG. 3. Intensity correlation function at detector F for $\mathcal{R}=0.01$ and (i) $\Gamma=0.4$; (ii) $\Gamma=0.5$; (iii) $\Gamma=0.6$; (iv) $\Gamma=0.8$; (v) $\Gamma=0.9$; (vi) $\Gamma=1.0$.

all rates are dimensionless numbers; $2\Gamma+2\bar{\Gamma}=2$ is the total spontaneous emission rate and $0\leq\Gamma\leq 1$.

For this example the non-Hermitian Hamiltonian is

$$\hat{\mathcal{H}} = i\hbar[\sqrt{\mathcal{R}K/2}(\hat{a}^\dagger - \hat{a}) - K\hat{a}^\dagger\hat{a} - \hat{\sigma}_+\hat{\sigma}_- - \sqrt{2K\Gamma}\hat{a}\hat{\sigma}_+] , \quad (11)$$

where the first term on the right-hand side represents a classical current driving the laser mode; $\hat{\sigma}_+$ and $\hat{\sigma}_-$ raise and lower the atom between states $|-\rangle$ (lower) and $|+\rangle$ (upper). There are now two kinds of collapses occurring at rates $R_F(t)=\langle\psi_c(t)|\hat{C}_F^\dagger\hat{C}_F|\psi_c(t)\rangle$ and $R_{\bar{F}}(t) =\langle\psi_c(t)|\hat{C}_{\bar{F}}^\dagger\hat{C}_{\bar{F}}|\psi_c(t)\rangle$, defined by the collapse operators

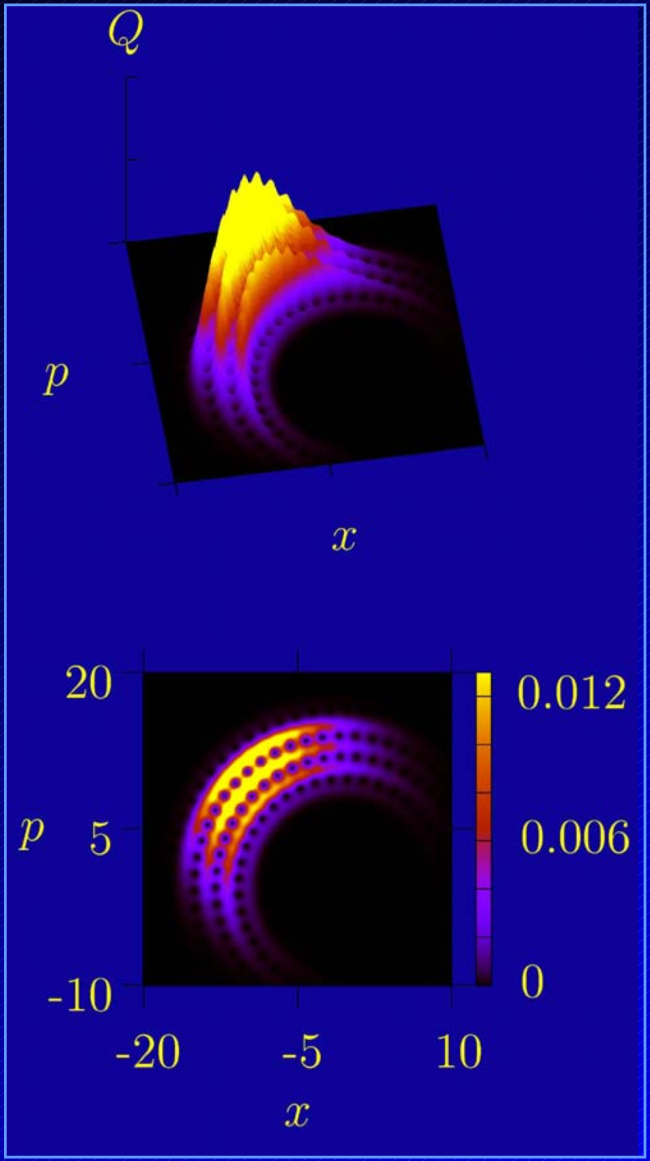
$$\hat{C}_F = \sqrt{2K}\hat{a} + \sqrt{\Gamma}\hat{\sigma}_-, \quad \hat{C}_{\bar{F}} = \sqrt{2-\Gamma}\hat{\sigma}_- . \quad (12)$$

If the cavity mode is initially in the vacuum state, the conditioned wave function factorizes in the form $|\psi_c(t)\rangle =|\alpha(t)\rangle|A_c(t)\rangle$, where $|\alpha(t)\rangle$ is a coherent state and $|A_c(t)\rangle$ is the state of the atom. After a short time

Using Eqs. (14), the corresponding photon detection rates (photon fluxes) are

$$R_F = \mathcal{R}(1+\mathcal{R}\Gamma)^{-1}(\bar{\Gamma}^2+\mathcal{R}\Gamma) ,$$
$$R_{\bar{F}} = \mathcal{R} - R_F . \quad (16)$$

Consider the case $\bar{\Gamma}=0$ which produces the largest bunching effect. In this case the incident light is focused within the atomic absorption cross section and we might expect a weak incident beam to be completely absorbed (reflected). Indeed, the transmitted photon flux is very small—$R_F\sim\mathcal{R}^2$ rather than $R_F\sim\mathcal{R}$. However, it is not zero; a few photons are transmitted. To understand why, and why these photons are highly bunched, we consider the wave-function collapse that accompanies the detection of a photon in transmission. Applying $\hat{C}_F$ to Eq. (15) gives



Entanglement between a coherent-state source and a qubit

PHYSICAL REVIEW A **68**, 046301 (2003)

Comment on "Some implications of the quantum nature of laser fields for quantum computations"

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A recent discussion of quantum limitations to the fidelity with which superpositions of internal atomic energy levels can be generated by an applied, quantized, laser pulse, is shown to be based on unrealistic physical assumptions. This discussion assumed the validity of Jaynes-Cummings dynamics for an atom interacting with a laser field in free space, that is, when the atom is not surrounded by a resonant cavity. If the laser field is a multimode quantum coherent state, and the Rabi frequency is much greater than the spontaneous decay rate, then the total atomic decoherence rate is on the order of the spontaneous decay rate. With the use of a unitary transformation of the field states due to Mollow, it can be shown that the atomic decoherence rate is the same as if the laser field were treated classically, without any additional contribution due to the quantum nature of the laser field.

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The quantum dynamics of a two-level atom in free space interacting with a resonant, coherent, quantized electromagnetic field is important from the standpoint of pure physics and potentially for practical applications. For example, some proposed implementations of quantum computation depend on the ability to accurately generate arbitrary superpositions of two atomic states by means of applied, resonant fields. If the external field is considered to be classical, then the

the field. All radiation emitted by the atom must go into that mode, and all radiation absorbed by the atom must come out of that mode. Thus, emitted radiation stays around and can be reabsorbed, and the absorption of radiation by the atom decreases the intensity of the applied field. The combination of these two effects leads to the complicated Jaynes-Cummings atomic dynamics, including the well-known collapses and revivals. The former effect (reabsorption of emit-

Atom-

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Some implications of the quantum nature of laser fields for quantum computations

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The quantum nature of the laser fields used in many proposed schemes for the manipulation of quantum information may have important consequences for some very large scale quantum computations. Some of these consequences are explored here, focusing especially on phase errors and their effects on error-correction schemes. Depending on the way the logical gates are performed, constraints are found on either the required number of photons per coherence time put out by the laser source, or the source's power

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I. INTRODUCTION AND MOTIVATION

Many schemes proposed for the manipulation of quantum information, on a wide variety of physical qubits, rely to some extent or another on laser pulses. While the fields of these pulses may typically be regarded as being classical to an excellent approximation, they are, in fact, quantum systems themselves, and as such they are subjected, in principle, to limitations such as the uncertainty principle and the possibility to become entangled with the physical qubits. The possible impact of these limitations on quantum computing has recently been the subject of some attention [1–3].

This paper focuses on the constraints arising from the requirement that the error rates due to the quantum nature of the field be sufficiently small for error-correction methods to

(In what follows this type of Hamiltonian will be taken as characteristic of "one-photon type" transitions.) It is easy to see that the desired transformation is accomplished by any pulse $E(t)$ of the form $E(t)=E_0(t)e^{i\phi}$ with E_0 real and $\phi=0$, such that

$$2\int_0^{\infty} gE_0(t)\,dt=\pi/2 \qquad (2)$$

(this is known as a $\pi/2$ pulse). If the pulse phase is $\phi\neq 0$, then the transformation accomplished is $|0\rangle\rightarrow(|0\rangle+e^{-i\phi}|1\rangle)/\sqrt{2}$, $|1\rangle\rightarrow(|0\rangle-e^{i\phi}|1\rangle)/\sqrt{2}$ instead. Of course, the phase of the pulse itself is arbitrary, but when one considers a sequence of pulses all acting on the same qubit, their

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quantify atom-field entanglement in various cases of interest. We find that the entanglement decreases with the average number of photons $\bar{n}$ in a laser beam as $E \propto \log_2 \bar{n}/\bar{n}$ for $\bar{n}\rightarrow\infty$.

I. INTRODUCTION

In many protocols for the implementation of quantum logic,

ount of atom-field entanglement in a given experiment is the focal area of the light beam. For instance, in an ion-trap quantum computer containing several ions each ion can in principle be addressed by focusing a laser beam onto the appropriate position. The focusing requirements are then obviously determined by the distances between neighboring ions. The same would apply to the situation where several atoms are kept inside optical cavities [6], for the purpose of quantum computation [7] or communication [8]. For a small array of qubits with

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Some implications of the quantum nature of laser fields for quantum computations

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The quantum nature of the laser fields used in many proposed schemes for the manipulation of quantum

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On the classical character of control fields in quantum information processing

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Control fields in quantum information processing are virtually always, almost by definition, assumed to be classical. In reality, however, when such a field is used to manipulate the quantum state of qubits, the qubits never remain completely unentangled with the field. For quantum information processing this is an undesirable property, as it precludes perfect quantum computing and quantum communication. Here we consider the interaction of atomic qubits with laser fields and quantify atom-field entanglement in various cases of interest. We find that the entanglement decreases with the average number of photons $\bar{n}$ in a laser beam as $E \propto \log_2 \bar{n}/\bar{n}$ for $\bar{n} \to \infty$.

I. INTRODUCTION

In many protocols for the implementation of quantum logic,

photon numbers because of the correspondence principle; for a classical field there would be no entanglement. Of course, if one were to use highly nonclassical states of the radiation field, such as photon number states, then this expectation would not be fulfilled, but for a laser beam well described by a mixture of number states with a Poissonian probability distribution, the entanglement indeed decreases with the average number of photons, as we will show here.

One important parameter that determines the amount of atom-field entanglement in a given experiment is the focal area of the light beam. For instance, in an ion-trap quantum computer containing several ions each ion can in principle be addressed by focusing a laser beam onto the appropriate position. The focusing requirements are then obviously determined by the distances between neighboring ions. The same would apply to the situation where several atoms are kept inside optical cavities [6], for the purpose of quantum computation [7] or communication [8]. For a small array of qubits with a mixture of number states with a Poissonian probability distribution, the entanglement indeed decreases with the average number of photons, as we will show here.

One important parameter that determines the amount of atom-field entanglement in a given experiment is the focal area of the light beam. For instance, in an ion-trap quantum computer containing several ions each ion can in principle be addressed by focusing a laser beam onto the appropriate position. The focusing requirements are then obviously determined by the distances between neighboring ions. The same would apply to the situation where several atoms are kept inside optical cavities [6], for the purpose of quantum computation [7] or communication [8]. For a small array of qubits with large spacings, the assumption of a classical laser field may indeed be justified. However, as the density of qubits increases, the external control field must be focused ever more tightly to avoid parasitic excitation of neighboring qubits. The question then arises whether the assumption of a classical field is justified for an atom localized on a wavelength scale with illumination of large numerical aperture. With such localization and illumination, the transmitted field might have imprinted upon it measurable signatures of its interaction with the atom. Such entanglement between atom and field would cause quantum information encoded in the atom to decohere. Of course there are avenues to mitigate this difficulty, as for example by focusing in a cylindrical geometry to increase the beam area while still keeping a small dimension along a linear array of atoms. Perhaps surprisingly, the general solution to this problem, e.g., for forward scattering and fluorescent fields is not known, even for the simple case of light focused onto a two-state atom. Relevant work includes the application of a standard input-output formalism to a quasi 1-dimensional version of this problem [9], and the construction of exact 3-dimensional vector solutions of the Maxwell equations, representing beams of light focused by a strong spherical lens [10], but these calculations do not directly address the question of entanglement. We attempt to fix that problem here.

We wish to assess the importance of decoherence (and its dependence on focusing parameters) due to atom-field entan-

the laser light.

The evolution operator $U(t) = \exp(-iHt/\hbar)$ is not easily evaluated explicitly in the general case. However, if the bandwidth B (the spread of frequencies) of the field is sufficiently small, a condition specified below, we can approximate the Hamiltonian by that of an atom in a fictituous single-mode coherent state with one frequency which we denote by ω_L. We tackle the problem of introducing the required approximations in two steps.

First consider the following simple Hamiltonian,

$$\tilde{H} = \hbar\Delta a^\dagger a + \hbar g(a^\dagger\sigma^- + \sigma^\dagger a), \tag{10}$$

with $\Delta = \omega_L - \omega_0$ the detuning from atomic resonance and a and $a^\dagger$ the annihilation and creation operators of the fictituous single-mode field. The Hamiltonian $\tilde{H}$ describes the well-known Jaynes-Cummings model. In fact, using this model the entanglement of a two-level atom interacting with a single-mode quantized field was studied in the early 90s within a very different context, namely, the occurrence of so-called collapses and revivals on very long time scales [16]. Here we are rather interested in short time scales. In fact the Jaynes-Cummings model would not even be valid in our case for longer times. For the Hamiltonian (10) an analytical solution of the evolution operator can be found easily [17]. In particular, expanding the time-dependent atom-field wave function as

$$|\Psi(t)\rangle = \sum_n c_0^n(t)|n\rangle|0\rangle + c_1^n(t)|n\rangle|1\rangle \tag{11}$$

we get

$$c_1^n(t) = \left(\left[\cos(\Omega_n t/2) - \frac{i\Delta}{\Omega_n}\sin(\Omega_n t/2)\right]c_1^n(0) - \frac{2ig\sqrt{n+1}}{\Omega_n}\sin(\Omega_n t/2)c_0^{n+1}(0)\right)\exp(i\Delta t/2),$$

atomic dynamics (Rabi oscillations), including decoherence due to technical imperfections in the classical driving field, can easily be calculated (see, e.g., Sec. 4 of Ref. [1]). In addition, decoherence due to radiative decay of the atomic states has been considered (see, e.g., Secs. 4.2.1 and 4.4.6.4 of Ref. [1]). Conceivably, the quantum nature of the driving field might lead to additional decoherence.

A recent attempt [2] to extend the calculations of Rabi oscillations in free space to the case of a quantized driving field used an inaccurate model, which is equivalent to a "reversed micromaser." That is, instead of an atom passing through a resonant cavity, an atom is intercepted by an electromagnetic field, confined to a region of space traveling at the speed of light. The context of this calculation was the necessity, in quantum computation, for high accuracy of quantum state control. Others have applied a more or less equivalent model to problems in quantum information processing [3]. In the "reversed micromaser" model, Fock states $|n\rangle$ apparently represent quantized field excitations confined to an imaginary box moving at the speed of light. While the atom is inside the field region, the atom-field state is presumed to follow Jaynes-Cummings dynamics [4]. In this model, a coherent laser pulse is represented by a superposition of moving Fock states $|\alpha\rangle = e^{-|\alpha|^2}\sum_{n=0}^{\infty}(\alpha^n/\sqrt{n!})|n\rangle$. Jaynes-Cummings dynamics then lead to entanglement of the atom and field and to effective decoherence of the atomic dynamics when a trace is performed over the field degrees of freedom.

This picture is unrealistic and inaccurate for an atom in free space, since there the field is not confined by a cavity. The physical problem with the Jaynes-Cummings model in free space is that it assumes that there is only one mode of

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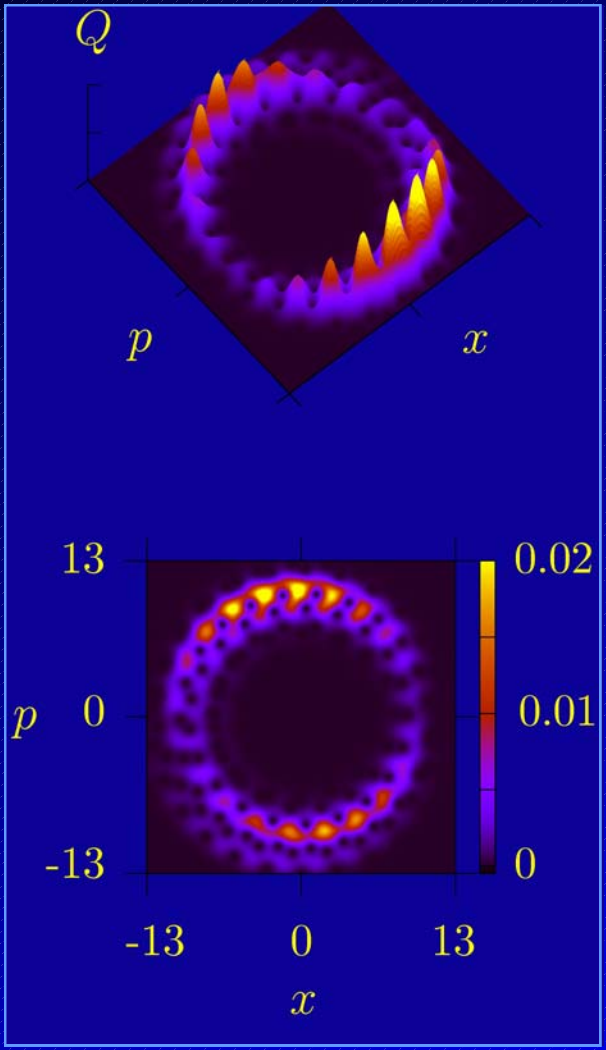
the field. All radiation emitted by the atom must go into that mode, and all radiation absorbed by the atom must come out of that mode. Thus, emitted radiation stays around and can be reabsorbed, and the absorption of radiation by the atom decreases the intensity of the applied field. The combination of these two effects leads to the complicated Jaynes-Cummings atomic dynamics, including the well-known collapses and revivals. The former effect (reabsorption of emitted radiation) does not occur in free space, because the emitted photon leaves the atom and does not interact with it again. The latter effect (a decrease in the applied field upon absorption of radiation by the atom) also does not occur in free space. It would correspond to a change in the laser pulse amplitude *upstream* from the atom. A change in the amplitude *downstream* does of course occur due to interference with the coherent forward-scattered field. Radiation is emitted by the atom in a dipole (or other multipole) pattern into all modes of the field and also as coherent forward scattering. Because the electromagnetic field has all modes available to it, not just a single one, the atomic dynamics will differ from those predicted by the Jaynes-Cummings model.

The Jaynes-Cummings model makes an odd prediction, which might be called the "beam area paradox." The Jaynes-Cummings (or "reversed micromaser") model predicts that the decoherence of the atomic system scales inversely with the mean number of photons $\langle n\rangle$ in the laser pulse. If one keeps the intensity at the site of the atom constant, but increases $\langle n\rangle$ by increasing the cross-sectional area of the beam, the decoherence is predicted to decrease. This has the appearance of being a *nonlocal* effect of the presence or absence of the field at arbitrarily large distances from the atom. This result is more explicit in the work of van Enk and Kimble [3], where the beam area A appears explicitly in, for



The cascaded open systems approach

H. Nha and H.J. Carmichael,
Phys. Rev. A 71, 013805-1–6 (2005)

couples to a classical driving field plus the quantized vacuum. Itano suggests that all entanglement originates in the interaction with the vacuum and is therefore resolved through spontaneous emission.

The first and single most important question arising from Itano's comment is whether or not the entanglement reported in [6,7] results in decoherence that is additional to, or included within, the decoherence rate obtained from the standard treatment of spontaneous emission for an atom driven by a classical field. The assertion (implication) of Refs. [6,7] is that it is additional to; Gea-Banacloche states so explicitly [9]: "… I wanted to focus, instead, on the decoherence due to the quantum nature of the laser field, which I took to be a *separate* source of error." Itano's position is that there is no decoherence in addition to spontaneous emission.

There is a second, more subtle question, featured most clearly in the reply of van Enk and Kimble [10]. Does the driven atom become entangled with the laser field at all? Considering, for sake of argument, that the total decoherence rate can be calculated as Itano claims, can any part of it be attributed to entanglement between the laser field and atom? By implication, if not directly, Itano claims such entanglement is zero. The authors of the criticized work claim it is not, though the entanglement is small [9,10], agreeing at most that the spontaneous emission calculation gives the correct number for the total decoherence rate, while asserting that its account of the laser-entanglement part of the decoherence is incorrect.

We return to these questions in Sec. VI, after presenting our own analysis of the problem of a two-state atom driven

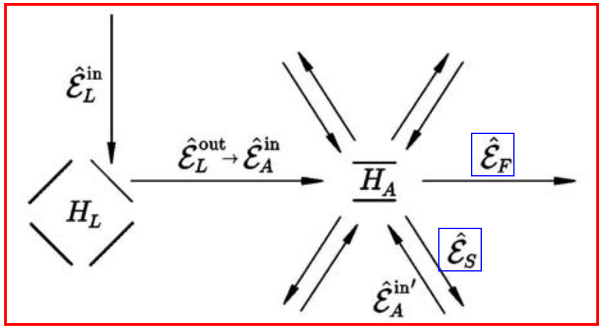


FIG. 1. Schematic diagram of the cascaded system of a two-state atom (Hamiltonian H_A) driven by a coherent laser source (Hamiltonian H_L). The various inputs and outputs are defined in the text.

laser subsystem, denoted L, and the target atom subsystem, denoted A. The subsystems have free Hamiltonians H_L and H_A, and couple through the quantized electromagnetic field, which is denoted as a reservoir with Hamiltonian H_R. Free fields $\hat{\mathcal{E}}_L^{\rm in}$ and $\hat{\mathcal{E}}_A^{\rm in\prime}$ provide vacuum inputs to subsystems L and A, respectively. Subsystems L and A couple unidirectionally through the common channel $\hat{\mathcal{E}}_L^{\rm in} \to \hat{\mathcal{E}}_L^{\rm out} \to \hat{\mathcal{E}}_A^{\rm in} \to \hat{\mathcal{E}}_F$. The scattered fields are the forward-scattered field $\hat{\mathcal{E}}_F$ and the sideways-scattered field $\hat{\mathcal{E}}_S$. All fields have units of the square root of photon flux.

A master equation for $L \otimes A$ is derived in the Born-

The Lindblad $\mathcal{L}_{\hat{J}_F}$ enters Eq. (5) through the coupling of L and A to the common (forward) scattering channel via the field $\mathcal{E}_L(t')$, where a decay rate $2\kappa_A$ is introduced phenomenologically to parametrize the strength of the coupling to the atom; this coupling strength depends on the overlap of the atomic dipole mode with the solid angle subtended by the laser pulse. The Lindblad $\mathcal{L}_{\hat{J}_S}$ accounts for the interaction of

source to the target atom is explicit. The last term, proportional to $\hat{a}\hat{\sigma}_+$, in Hamiltonian (10) annihilates a laser photon and excites the atom, but there is no term $\hat{a}^\dagger\hat{\sigma}_-$ for the re-emission of photons into the laser field. Thus, as Itano pointed out, interaction with the atom does not change the laser field *upstream* from the atom (the time-retarded field at the source). The atom does, however, absorb photons emitted a retardation time earlier by the laser source. Photons are couples to a classical driving field plus the quantized vacuum. Itano suggests that all entanglement originates in the interaction with the vacuum and is therefore resolved through spontaneous emission.

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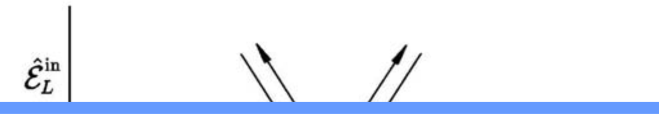


$$U_L(l/c) \equiv \exp[(i/\hbar)(H_L + H_R + H_{LR})(l/c)], \tag{4}$$

where H_{LR} is the interaction between L and R. Then from the reservoir couplings noted above, applied formally at $t'=0$, by a standard derivation, the equation of motion for the reduced density operator $\rho' \equiv \mathrm{tr}_R(\chi')$ is [11–13]

$$\dot{\rho}' = \frac{1}{i\hbar}[H_0, \rho'] + \mathcal{L}_{\hat{J}_F}\rho' + \mathcal{L}_{\hat{J}_S}\rho', \tag{5}$$

where $\mathcal{L}_{\hat{O}}$ is the Lindblad superoperator,

$$\mathcal{L}_{\hat{O}} \equiv \hat{O}\cdot\hat{O}^\dagger - \frac{1}{2}\hat{O}^\dagger\hat{O}\cdot - \frac{1}{2}\cdot\hat{O}^\dagger\hat{O}, \tag{6}$$

and

$$H_0 = H_L + H_A + i\hbar\sqrt{\kappa_L\kappa_A}(\hat{a}^\dagger\hat{\sigma}_- - \hat{a}\hat{\sigma}_+), \tag{7}$$

where $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are raising and lowering operators for the atom. The forward- and side-scattering jump operators are

$$\boxed{\hat{J}_F = \sqrt{2\kappa_L}\hat{a} + \sqrt{2\kappa_A}\hat{\sigma}_-,} \quad \boxed{\hat{J}_S = \sqrt{2\kappa_A'}\hat{\sigma}_-.} \tag{8}$$

The Lindblad $\mathcal{L}_{\hat{J}_F}$ enters Eq. (5) through the coupling of L and A to the common (forward) scattering channel via the field $\mathcal{E}_L(t')$, where a decay rate $2\kappa_A$ is introduced phenomenologically to parametrize the strength of the coupling to the atom; this coupling strength depends on the overlap of the atomic dipole mode with the solid angle subtended by the laser pulse. The Lindblad $\mathcal{L}_{\hat{J}_S}$ accounts for the interaction of

$$\boxed{H_B = H_L + H_A - i\hbar\kappa_L\hat{a}^\dagger\hat{a} - i\hbar\frac{\gamma}{2}\hat{\sigma}_+\hat{\sigma}_- - 2i\hbar\sqrt{\kappa_L\kappa_A}\hat{a}\hat{\sigma}_+,} \tag{10}$$

and suffers quantum jumps,

$$|\overline{\psi_{\rm REC}}\rangle \to \hat{J}_F|\overline{\psi_{\rm REC}}\rangle, \tag{11a}$$

$$|\overline{\psi_{\rm REC}}\rangle \to \hat{J}_S|\overline{\psi_{\rm REC}}\rangle, \tag{11b}$$

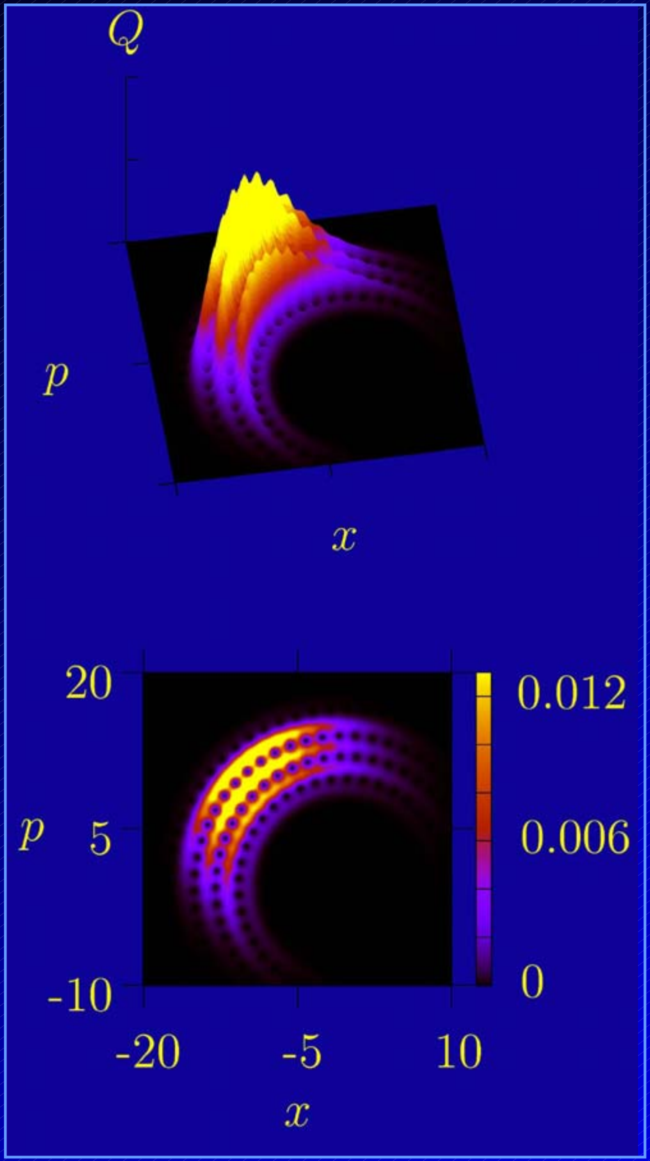
determined in a Monte Carlo fashion with probabilities (per time step dt)

$$p_{LA} = \langle\psi_{\rm REC}|\hat{J}_F^\dagger\hat{J}_F|\psi_{\rm REC}\rangle dt, \tag{12a}$$

$$p_A = \langle\psi_{\rm REC}|\hat{J}_S^\dagger\hat{J}_S|\psi_{\rm REC}\rangle dt. \tag{12b}$$

Jumps executed by the operators $\hat{J}_F$ and $\hat{J}_S$ denote the detection of a photon in the forward- and side-scattered fields, respectively ($\hat{\mathcal{E}}_F$ and $\hat{\mathcal{E}}_S$ in Fig. 1).

In this formulation the unidirectional coupling of the laser source to the target atom is explicit. The last term, proportional to $\hat{a}\hat{\sigma}_+$, in Hamiltonian (10) annihilates a laser photon and excites the atom, but there is no term $\hat{a}^\dagger\hat{\sigma}_-$ for the re-emission of photons into the laser field. Thus, as Itano pointed out, interaction with the atom does not change the laser field *upstream* from the atom (the time-retarded field at the source). The atom does, however, absorb photons emitted a retardation time earlier by the laser source. Photons are



Itano … the "beam area paradox"

addition, decoherence due to radiative decay of the atomic states has been considered (see, e.g., Secs. 4.2.1 and 4.4.6.4 of Ref. [1]). Conceivably, the quantum nature of the driving field might lead to additional decoherence.

A recent attempt [2] to extend the calculations of Rabi oscillations in free space to the case of a quantized driving field used an inaccurate model, which is equivalent to a "reversed micromaser." That is, instead of an atom passing through a resonant cavity, an atom is intercepted by an electromagnetic field, confined to a region of space traveling at the speed of light. The context of this calculation was the necessity, in quantum computation, for high accuracy of quantum state control. Others have applied a more or less equivalent model to problems in quantum information processing [3]. In the "reversed micromaser" model, Fock states $|n\rangle$ apparently represent quantized field excitations confined to an imaginary box moving at the speed of light. While the atom is inside the field region, the atom-field state is presumed to follow Jaynes-Cummings dynamics [4]. In this model, a coherent laser pulse is represented by a superposition of moving Fock states $|\alpha\rangle = e^{-|\alpha|^2}\Sigma_{n=0}^{\infty}(\alpha^n/\sqrt{n!})|n\rangle$. Jaynes-Cummings dynamics then lead to entanglement of the atom and field and to effective decoherence of the atomic dynamics when a trace is performed over the field degrees of freedom.

This picture is unrealistic and inaccurate for an atom in free space, since there the field is not confined by a cavity. The physical problem with the Jaynes-Cummings model in free space is that it assumes that there is only one mode of

free space. It would correspond to a change in the laser pulse amplitude *upstream* from the atom. A change in the amplitude *downstream* does of course occur due to interference with the coherent forward-scattered field. Radiation is emitted by the atom in a dipole (or other multipole) pattern into all modes of the field and also as coherent forward scattering. Because the electromagnetic field has all modes available to it, not just a single one, the atomic dynamics will differ from those predicted by the Jaynes-Cummings model.

The Jaynes-Cummings model makes an odd prediction, which might be called the "beam area paradox." The Jaynes-Cummings (or "reversed micromaser") model predicts that the decoherence of the atomic system scales inversely with the mean number of photons $\langle n\rangle$ in the laser pulse. If one keeps the intensity at the site of the atom constant, but increases $\langle n\rangle$ by increasing the cross-sectional area of the beam, the decoherence is predicted to decrease. This has the appearance of being a *nonlocal* effect of the presence or absence of the field at arbitrarily large distances from the atom. This result is more explicit in the work of van Enk and Kimble [3], where the beam area A appears explicitly in, for example, Eq. (31), and where they state, "Decreasing the focal area A will increase the amount of entanglement."

If the applied laser field is treated classically, but a phenomenological decay rate γ for the upper level is included, one finds that the atomic decoherence rate is on the order of γ if the field is strong. "Strong" here means that the time required for the atom to undergo an induced transition (Rabi oscillation) is much less than the spontaneous lifetime of the

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filled circles indicate numerical results for times $\tau_n = 2^{-n}\pi$ for $n = 1\ldots16$, while the solid curves correspond to the approximate analytic solution (28).

1. Entanglement in typical experiments

So how much do atom and field become entangled in a

It is perhaps also interesting to compare these numbers to those for a dipole transition under similar circumstances. For instance, for the $6S_{1/2}$ to $6P_{3/2}$ dipole transition in Cs (at a wavelength $\lambda \approx 850$nm, and an upper state lifetime of $\tau_0 \approx 31$ns), at the same laser power P and the same focusing area A, one would have a duration of $T = 0.46$ns for a NOT operation, an entanglement of $E = 7.6 \times 10^{-5}$ and a spontaneous emission probability during the NOT operation addition, decoherence due to radiative decay of the atomic states has been considered (see, e.g., Secs. 4.2.1 and 4.4.6.4 of Ref. [1]). Conceivably, the quantum nature of the driving field might lead to additional decoherence.

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absorption of radiation by the atom) also does not occur in free space. It would correspond to a change in the laser pulse amplitude *upstream* from the atom. A change in the amplitude *downstream* does of course occur due to interference with the coherent forward-scattered field. Radiation is emitted by the atom in a dipole (or other multipole) pattern into all modes of the field and also as coherent forward scattering. Because the electromagnetic field has all modes available to it, not just a single one, the atomic dynamics will differ from

This expression describes the entanglement well even when τ is not small but $\tau^2/|\alpha|^2$ is.

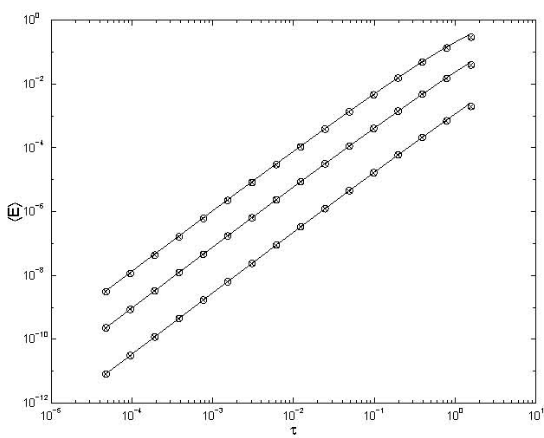

FIG. 2. Average entanglement as a function of scaled time τ for different values of the average photon number $\bar{n} = |\alpha|^2$; from top to bottom curves we have $|\alpha|^2 = 2^3, 2^7, 2^{12}$. The filled circles indicate numerical results for times $\tau_n = 2^{-n}\pi$ for $n = 1 \ldots 16$, while the solid curves correspond to the approximate analytic solution (28).

1. Entanglement in typical experiments

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$A \approx 100(\mu\mathrm{m})^2$, a typical quadrupole moment of $Q \approx ea_0^2$ [19] and a power of $P = 100\mu$W, yields for a wavelength of 730nm (corresponding to the $\mathrm{S}_{1/2}$ to $\mathrm{D}_{5/2}$ transition in $^{40}\mathrm{Ca}^+$ [18]) $T \approx 3.1\mu$s and $|\alpha|^2 \approx 1.1 \times 10^9$, so that

$$E \approx 2.2 \times 10^{-8}.$$

Decreasing the focal area A will increase the amount of entanglement. But even very strong focusing to areas of size $A \approx \lambda^2$ still does not lead to large entanglement. In fact, if we make A smaller by a factor 100 so that $A \approx 1(\mu\mathrm{m})^2$ (although we should note that the 1-dimensional model of Eq. (4) would cease to be valid for such small values of A) and decrease the power by a factor of 100 as well such that T remains constant, the entanglement increases by about a factor of 100 to a value $E \approx 10^{-6}$ that is still very small.

It is interesting to compare the smallness of the entanglement to the probability of spontaneous emission. Here the lifetime τ_0 of the metastable $\mathrm{D}_{5/2}$ state is about 1 sec. Since the interaction time is $T = 3.1\mu$ sec, the spontaneous emission probability during a NOT operation is thus $p_{\rm spon} = T/(2\tau_0) = 1.6 \times 10^{-6}$ (the factor 1/2 arises since the atom spends half of the time in the excited state).

It is perhaps also interesting to compare these numbers to those for a dipole transition under similar circumstances. For instance, for the $6\mathrm{S}_{1/2}$ to $6\mathrm{P}_{3/2}$ dipole transition in Cs (at a wavelength $\lambda \approx 850$nm, and an upper state lifetime of $\tau_0 \approx 31$ns), at the same laser power P and the same focusing area A, one would have a duration of $T = 0.46$ns for a NOT operation, an entanglement of $E = 7.6 \times 10^{-5}$ and a spontaneous emission probability during the NOT operation all rates are dimensionless numbers; $2\Gamma + 2\bar{\Gamma} = 2$ is the total spontaneous emission rate and $0 \le \Gamma \le 1$.

For this example the non-Hermitian Hamiltonian is

$$\boxed{\begin{aligned}\hat{\mathcal{H}} = i\hbar[\sqrt{\mathcal{R}K/2}(\hat{a}^\dagger - \hat{a}) - \kappa\hat{a}^\dagger\hat{a} \\ - \hat{\sigma}_+\hat{\sigma}_- - \sqrt{2K\Gamma}\hat{a}\hat{\sigma}_+]\end{aligned}}\,, \tag{11}$$

where the first term on the right-hand side represents a classical current driving the laser mode; $\hat{\sigma}_+$ and $\hat{\sigma}_-$ raise and lower the atom between states $|-\rangle$ (lower) and $|+\rangle$ (upper). There are now two kinds of collapses occurring at rates $R_F(t) = \langle\psi_c(t)|\hat{C}_F^\dagger\hat{C}_F|\psi_c(t)\rangle$ and $R_{\bar{F}}(t) = \langle\psi_c(t)|\hat{C}_{\bar{F}}^\dagger\hat{C}_{\bar{F}}|\psi_c(t)\rangle$, defined by the collapse operators

$$\boxed{\hat{C}_F = \sqrt{2K}\hat{a} + \sqrt{\Gamma}\hat{\sigma}_-,} \quad \hat{C}_{\bar{F}} = \sqrt{2-\Gamma}\hat{\sigma}_- . \tag{12}$$

If the cavity mode is initially in the vacuum state, the conditioned wave function factorizes in the form $|\psi_c(t)\rangle = |\alpha(t)\rangle|A_c(t)\rangle$, where $|\alpha(t)\rangle$ is a coherent state and $|A_c(t)\rangle$ is the state of the atom. After a short time $\alpha(t) \to \alpha_{ss} = \sqrt{\mathcal{R}/2K}$. Then the quantum trajectory for the atom is governed by the Schrödinger equation and collapse operators

$$|\dot{\bar{A}}_c\rangle = -(\hat{\sigma}_+\hat{\sigma}_- + \sqrt{\mathcal{R}\Gamma}\hat{\sigma}_+)|\bar{A}_c\rangle , \tag{13}$$

$$\hat{C}_F = \sqrt{\mathcal{R}} + \sqrt{\Gamma}\hat{\sigma}_-, \quad \hat{C}_{\bar{F}} = \sqrt{2-\Gamma}\hat{\sigma}_- . \tag{14}$$

Equations (13) and (14) are equivalent to those for an atom *inside* a coherently driven cavity in the bad-cavity

Using Eqs. (14), the corresponding photon detection rates (photon fluxes) are

$$\begin{aligned} R_F &= \mathcal{R}(1+\mathcal{R}\Gamma)^{-1}(\bar{\Gamma}^2 + \mathcal{R}\Gamma) , \\ R_{\bar{F}} &= \mathcal{R} - R_F . \end{aligned} \tag{16}$$

Consider the case $\bar{\Gamma} = 0$ which produces the largest bunching effect. In this case the incident light is focused within the atomic absorption cross section and we might expect a weak incident beam to be completely absorbed (reflected). Indeed, the transmitted photon flux is very small—$R_F \sim \mathcal{R}^2$ rather than $R_F \sim \mathcal{R}$. However, it is not zero; a few photons are transmitted. To understand why, and why these photons are highly bunched, we consider the wave-function collapse that accompanies the detection of a photon in transmission. Applying $\hat{C}_F$ to Eq. (15) gives

$$|A_c\rangle = (\bar{\Gamma}^2 + \mathcal{R}\Gamma)^{-1/2}(\bar{\Gamma}|-\rangle - \sqrt{\mathcal{R}\Gamma}|+\rangle) , \tag{17}$$

and the new detection rates

$$\begin{aligned} R_{F|F} &= \mathcal{R}(\bar{\Gamma}^2 + \mathcal{R}\Gamma)^{-1}[(\bar{\Gamma} - \Gamma)^2 + \mathcal{R}\Gamma] , \\ R_{\bar{F}|F} &= \mathcal{R} - R_F + \mathcal{R}(\bar{\Gamma}^2 + \mathcal{R}\Gamma)^{-1}2\Gamma^2 . \end{aligned} \tag{18}$$

For $\bar{\Gamma} = 0$ the forward photon flux is now $\Gamma = 1$, a change $R_{F|F}/R_F \sim 1/\mathcal{R}^2$. This huge increase in flux produces the

tion and the second is detected in the backward direction.

To conclude let me answer a question that might seem, superficially, to raise doubts about the new theory. It is usual to model the interaction between an atom and coherent light in a time-symmetric way, with an interaction Hamiltonian that can raise *and* lower the atom. $\hat{\mathcal{H}}$ [Eq. (11)] only includes the raising part; it is reasonable, then, to ask: How is it possible for a Rabi oscillation to occur? The answer lies in the collapse operator $\hat{C}_F$ [Eq. (14)]. This accounts for transitions that lower the atom while a photon is emitted into the coherent beam that excites the atom. Consider the limit $\mathcal{R} \to \infty$, $\Gamma \to 0$, with $\sqrt{\mathcal{R}\Gamma} = \Omega/2$, where Ω is the Rabi frequency. In the ap-

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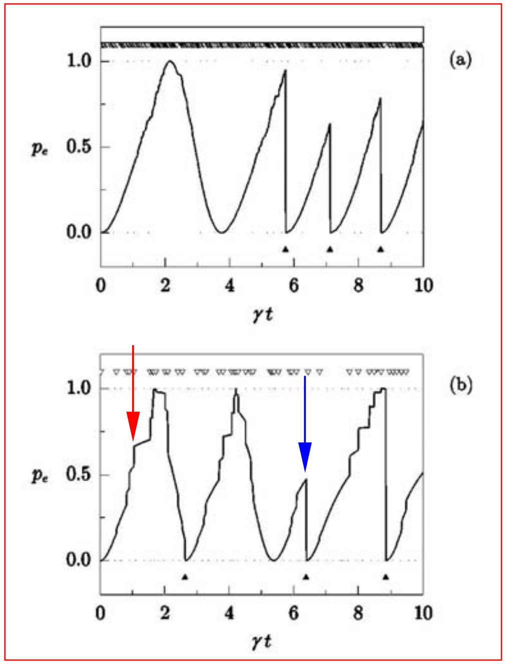


FIG. 2. Sample Monte Carlo simulations of the Rabi oscillation of the driven two-state atom in the presence of spontaneous emission, total emission rate $\gamma = 2\kappa_A + 2\kappa_A'$; for a Rabi frequency $\Omega = \sqrt{\kappa_L \kappa_A}|\alpha| = 2\gamma$ (for coherent state $|\alpha\rangle_L$) and forward-scattering rate (a) $2\kappa_A/\gamma = 0.04$, (b) $2\kappa_A/\gamma = 0.4$. The probability $p_e(t)$ to find the atom in the excited state is plotted as a function of time. Open (closed) triangles mark the times of forwards-scattering (side-scattering) quantum jumps.

does not put the atom in its ground state. In fact, the unique form of the $\hat{J}_F$ jump is of interest for a separate reason. In the absence of a term $\hat{a}^\dagger \hat{\sigma}_-$ in the Hamiltonian H_B, we might

scattering out of the forward channel. The forward part, moreover, is subject to a $1/\bar{n}$ effect as the previous authors have claimed [6,7,9,10].

V. MASTER EQUATION FOR THE ATOM

Considering the different forms of the quantum jumps in the two channels, it is unclear whether the total decoherence rate may be considered to be due to spontaneous emission alone or not. To resolve this issue we set the quantum trajectory formulation aside and derive a master equation for the atom alone.

We note first, from Eq. (5), that the equation of motion for the reduced density operator of L, $\rho_L = \mathrm{tr}_A(\rho')$, is given by the laser master equation

$$\dot{\rho}_L = \frac{1}{i\hbar}[H_L, \rho_L] + \mathcal{L}_{\hat{J}_L}\rho_L, \qquad (14)$$

with

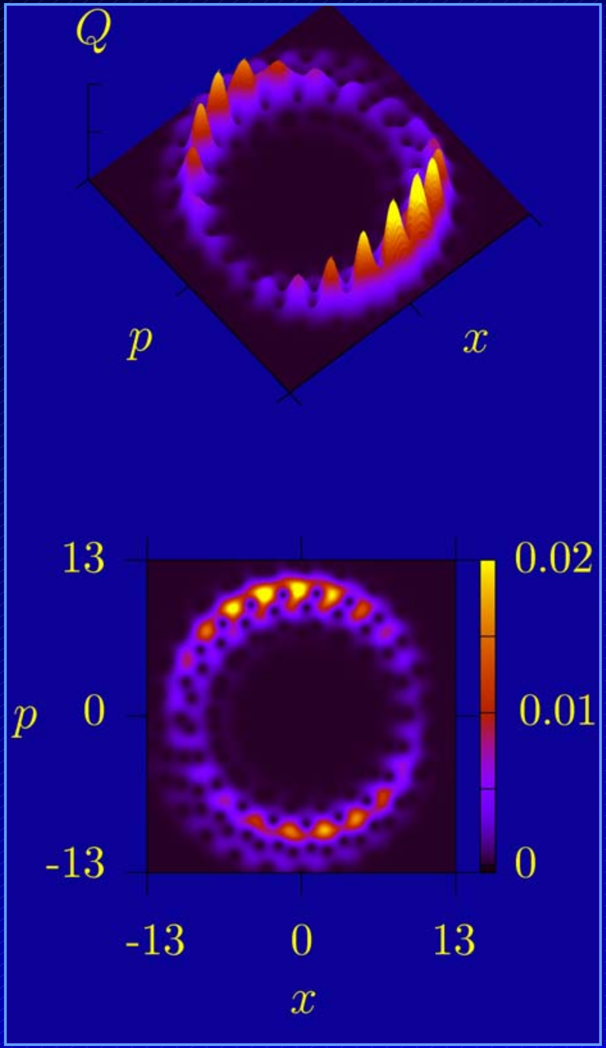
$$\hat{J}_L = \sqrt{2\kappa_A}\hat{a}. \qquad (15)$$

To generate a coherent state of the intracavity field, we adopt the model Hamiltonian

$$H_L = \hbar\omega_L \hat{a}^\dagger \hat{a} + i\hbar\kappa_L[\lambda(t)e^{-i\omega_L t}\hat{a}^\dagger - \lambda^*(t)e^{i\omega_L t}\hat{a}], \qquad (16)$$

where ω_L is the cavity resonant frequency and $\lambda(t)$ is the complex amplitude of a time-dependent classical current driving the cavity mode. In a rotating frame, with frequency ω_L, it is readily shown (assuming the initial state to be coherent) that the intracavity field is in the coherent state $|\alpha(t)\rangle_L$, with $\alpha(t)$ satisfying the equation

$$\dot{\alpha}(t) = \kappa_A\lambda(t) - \kappa_A\alpha(t). \qquad (17)$$



Entanglement between a laser source and a qubit

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QUANTUM THEORY OF AN OPTICAL MASER*

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The theory of an optical maser due to Lamb[1-3] is generally accepted as giving a realistic account of laser oscillation. The laser radiation was described by means of classical electrodynamics while the atoms were treated quantum mechanically. In this way, phenomena such as frequency pulling, variation of intensity with cavity tuning, mode competition, etc., were successfully described. There has been considerable interest recently in a quantum theory of laser behavior. It is the purpose of this Letter to give an account of such a theory.

To simplify the presentation we consider only single-mode oscillation and ignore the effects of atomic motion and spatial variation of the

laser radiation.

The states a and b of the atom are assumed to decay as in the Wigner-Weisskopf theory of radiation damping. For the state a, we introduce a group of states c, s where c is a level to which the atom decays with the emission of (nonlaser) radiation of type s. Similarly b decays to d, σ; the decay constants are denoted by γ_a and γ_b, respectively.

To obtain $\rho_{n,n'}(t_0+T)$ we must follow the time development of the combined atom-field system until the atom has decayed, and then trace over the states c, s and d, σ. We may obtain the rate of change of the density matrix due to many atoms, injected at random times

time characterizing the growth or decay of the

following equations of motion for the laser radiation (written in the Schrödinger picture):

$$\dot\rho_{n,n'} = -i(n-n')\Omega\rho_{n,n'} - [(n+1)R_{n,n'} + (n'+1)R_{n',n}^{*}]\rho_{n,n'} + [R_{n-1,n'-1} + R_{n'-1,n-1}^{*}] \times (nn')^{1/2}\rho_{n-1,n'-1} - \tfrac{1}{2}(\nu/Q)(n+n')\rho_{n,n'} + (\nu/Q)[(n+1)(n'+1)]^{1/2}\rho_{n+1,n'+1}, \qquad (2)$$

$$a(t)\,|\,n,-\rangle + b(t)|n+1,+\rangle$$