Controlling the mechanical state of an atom

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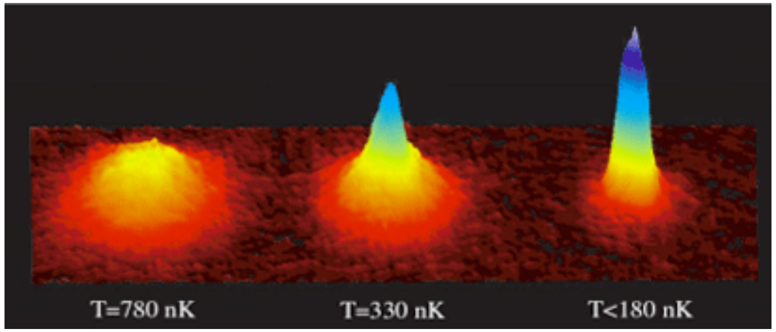
Cooling is ~~routine~~ ubiquitous



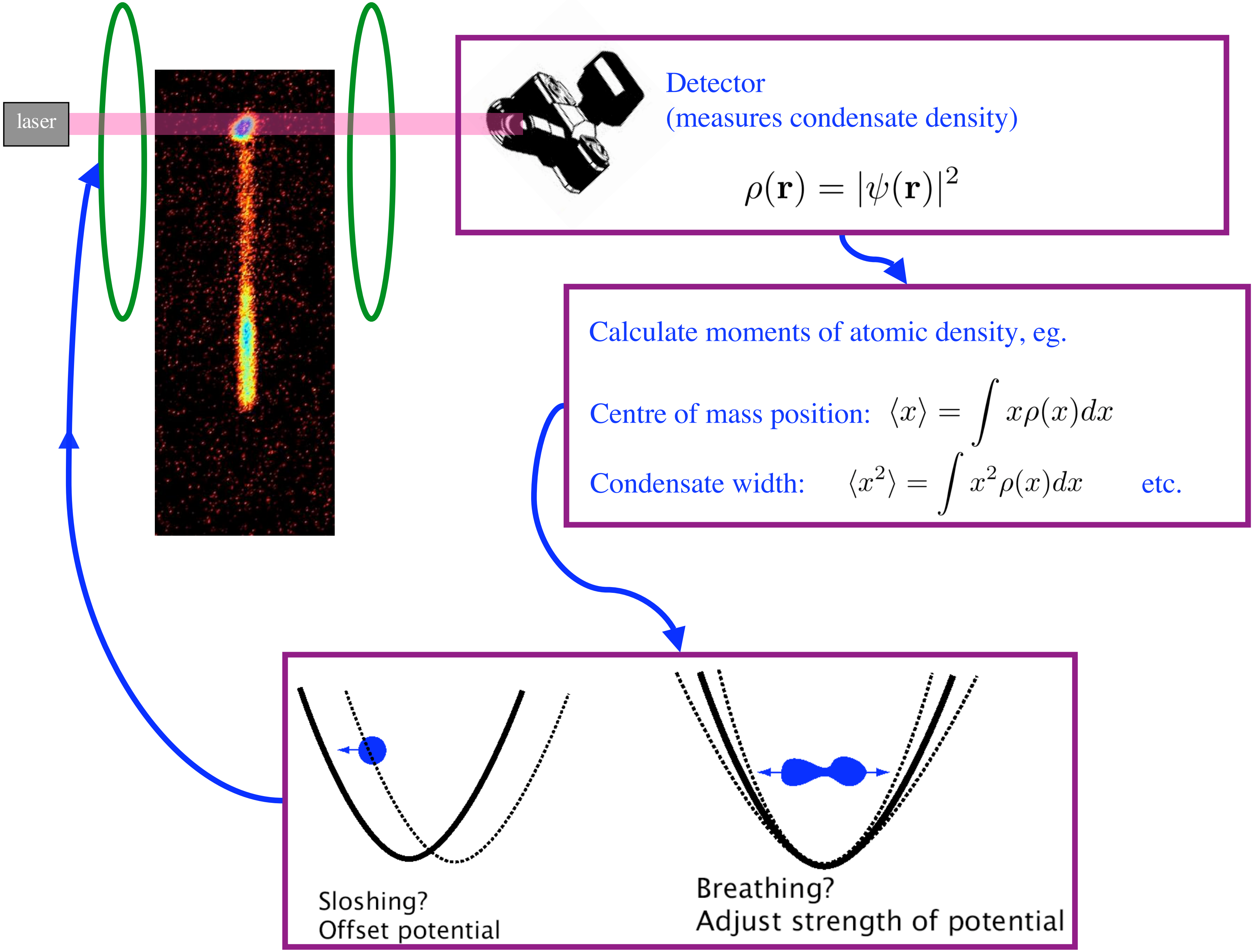
But still… it moves

- BECs are routinely created in excited states

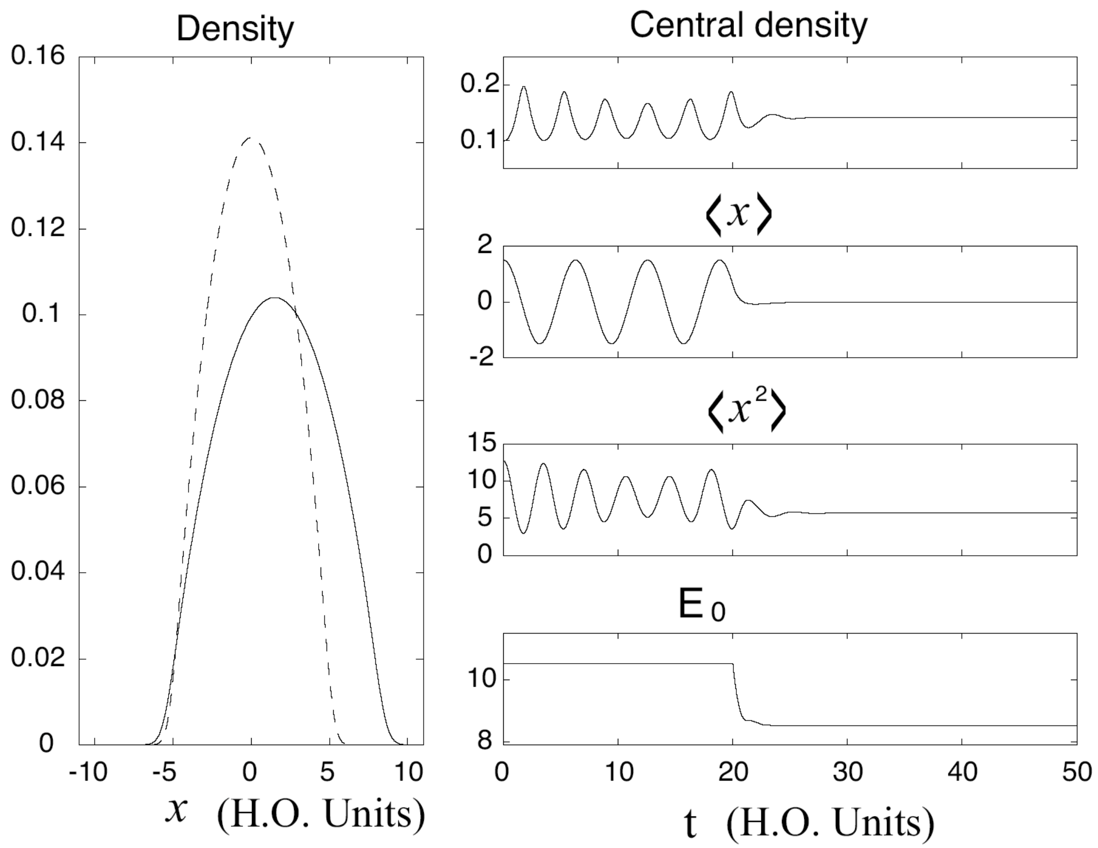
- Excitations are most obvious in atom laser output

- Pumping will exacerbate this issue

- Stability can be achieved passively or actively

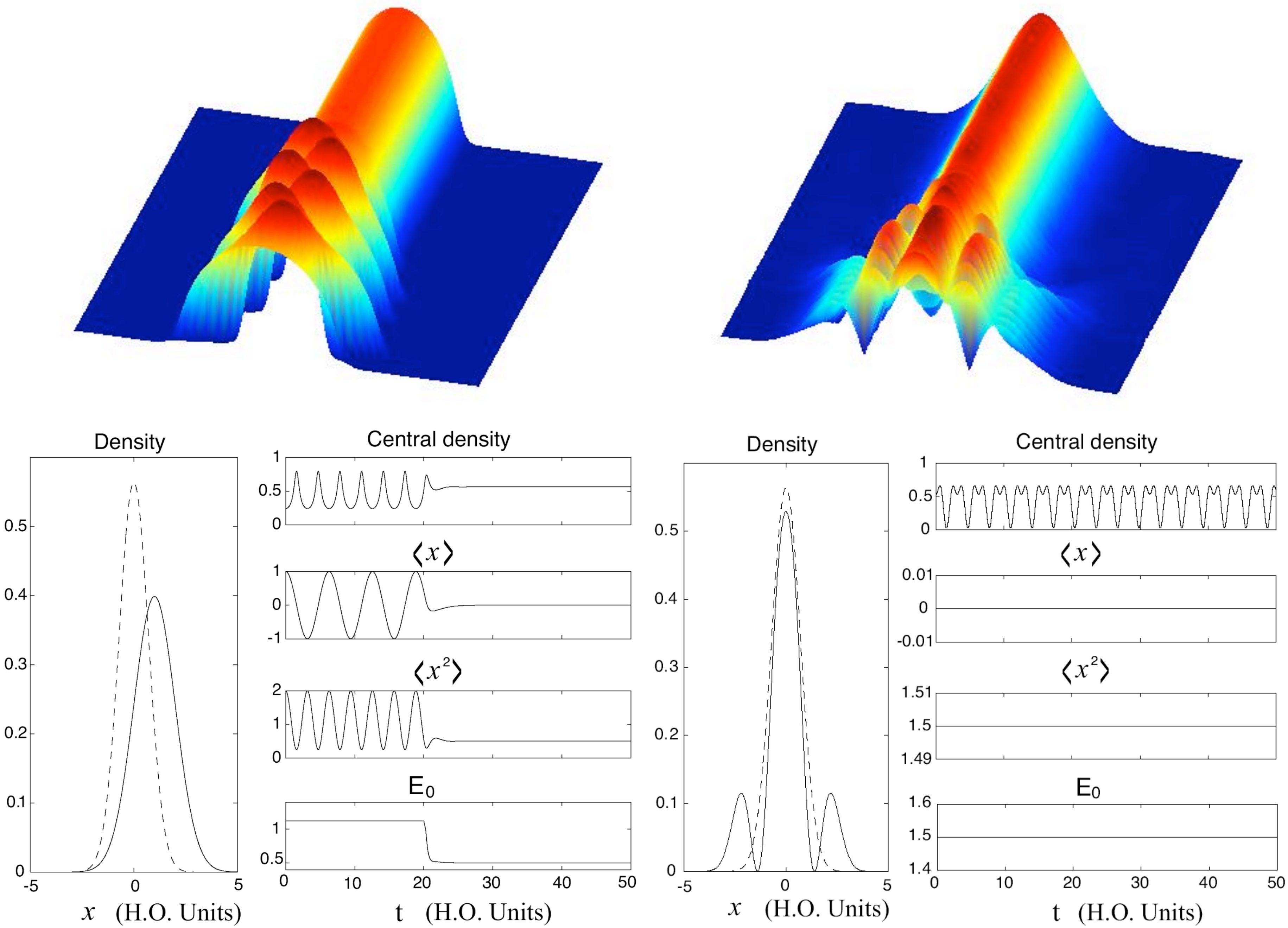

laser
Detector
(measures condensate density)
$\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$
Calculate moments of atomic density, eg.
Centre of mass position: $\langle x \rangle = \int x \rho(x) dx$
Condensate width: $\langle x^2 \rangle = \int x^2 \rho(x) dx$ etc.
Sloshing?
Offset potential
Breathing?
Adjust strength of potential

Semiclassical feedback to a BEC



PRA **69,** 013605 (2004)

Fast early effect, slow leakage induced by nonlinearities



Density
Central density
$\langle x \rangle$
$\langle x^2 \rangle$
E_0
x (H.O. Units)
t (H.O. Units)
Density
Central density
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$\langle x^2 \rangle$
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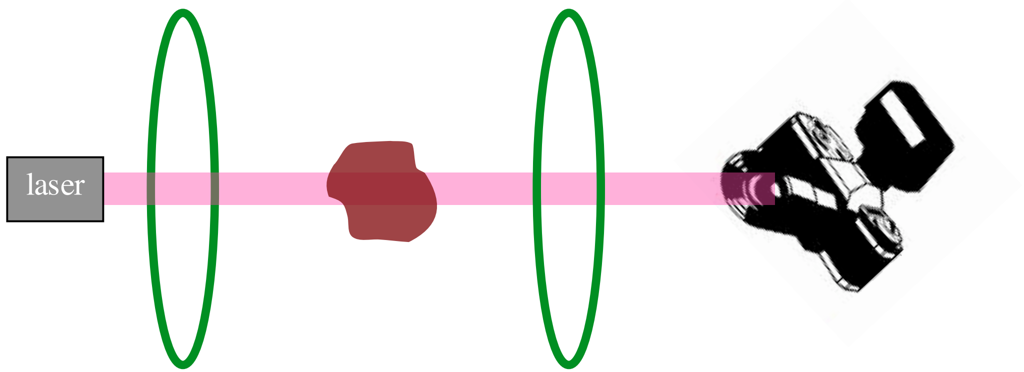


$$H_A = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 - u(t)q$$

Effect of the measurement:

$$d\rho = \frac{-i}{\hbar}[H_A, \rho]\ dt + \alpha\mathcal{D}[q]\rho\, dt$$

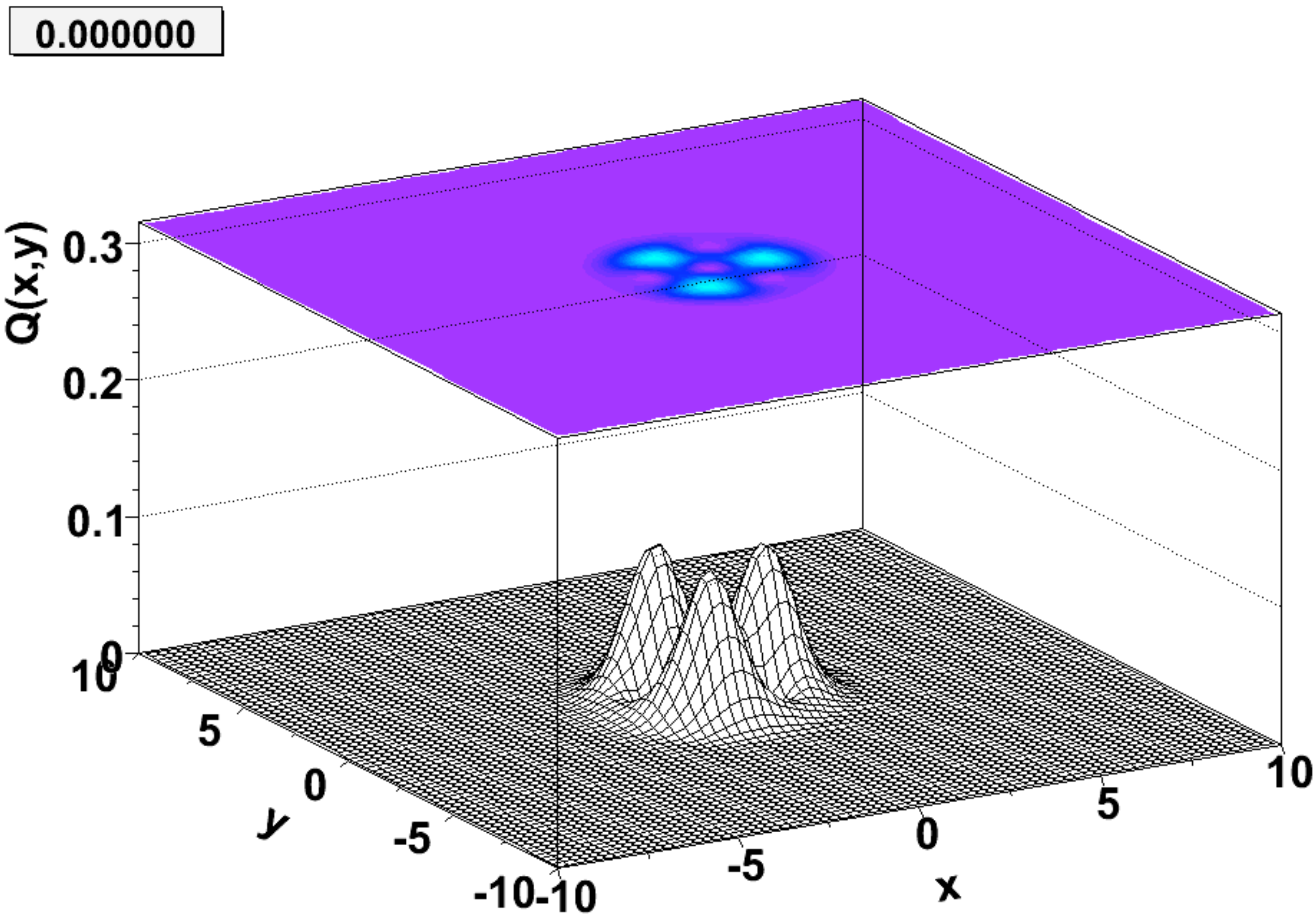
Conditional state (on a specific measurement):

$$d\pi_t = \frac{-i}{\hbar}[H_A, \pi_t]\ dt + \alpha\mathcal{D}[q]\pi_t dt + \sqrt{\eta\alpha}\left(q\pi_t + \pi_t q - 2\pi_t\,\mathrm{Tr}[q\pi_t]\right)\ dW$$

Solved optimally for Gaussian states (Gaussian initial state is preserved)

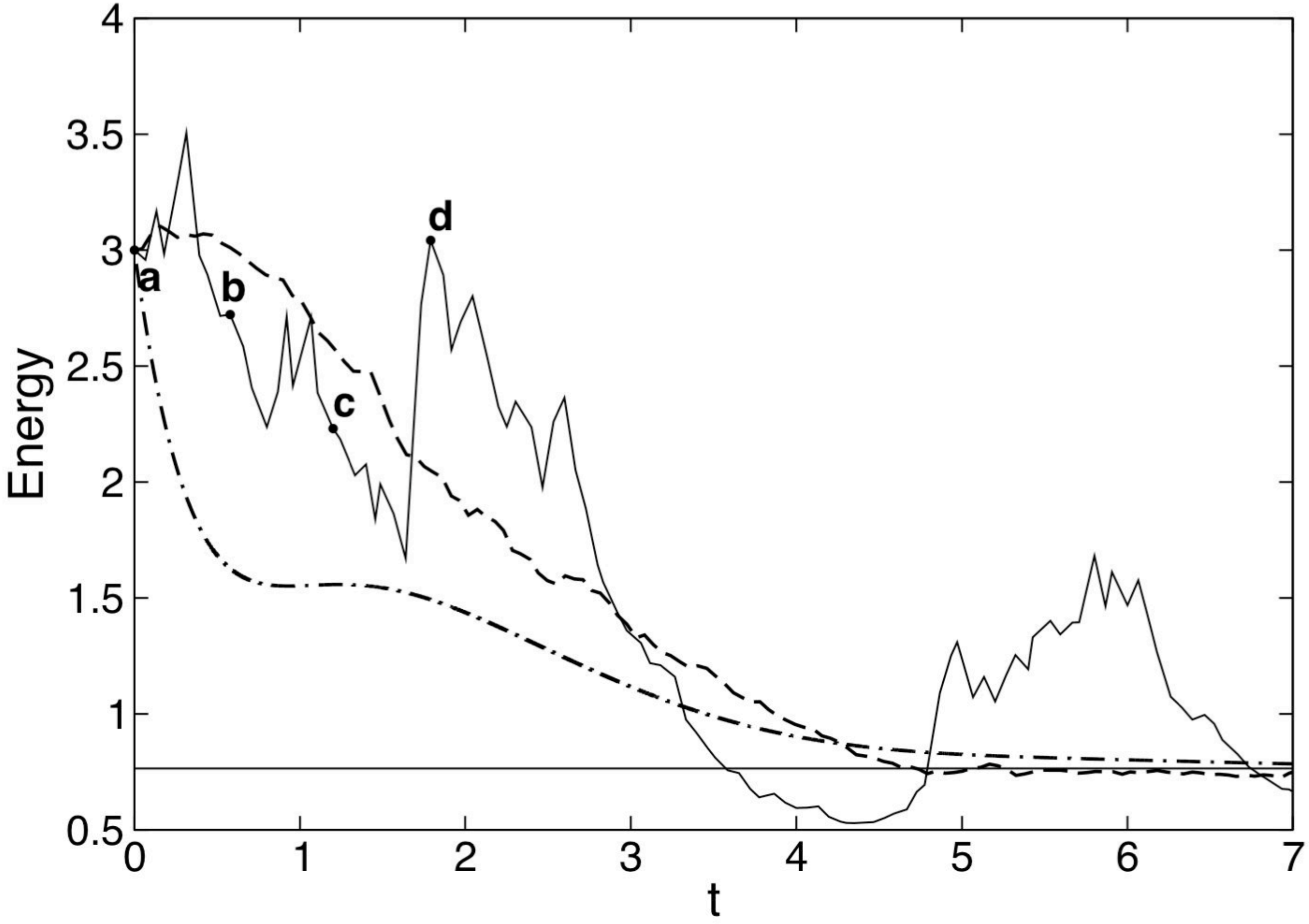
Doherty and Jacobs, PRA 60, 2700 (1999).

$$
\begin{aligned}
dQ(x,y,t) \quad = \quad & \left(\omega\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) - k_p\left(\int\int dx'dy'\; y'Q(x',y',t)\right)\frac{\partial}{\partial y} + \frac{\hbar\alpha}{4m\omega}\frac{\partial^2}{\partial y^2}\right)Q(x,y,t)\;dt \\
& +2\alpha\sqrt{\frac{\hbar\eta}{2m\omega}}\left(4x + \frac{\partial}{\partial x} - 4\left(\int\int dx'dy'\; x'Q(x',y',t)\right)\right)Q(x,y,t)\;dW
\end{aligned}
$$



$$dQ(x,y,t) = \left(\omega\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) - k_p\left(\int\int dx'dy'\; y'Q(x',y',t)\right)\frac{\partial}{\partial y} + \frac{\hbar\alpha}{4m\omega}\frac{\partial^2}{\partial y^2}\right)Q(x,y,t)\,dt$$

$$+2\alpha\sqrt{\frac{\hbar\eta}{2m\omega}}\left(4x + \frac{\partial}{\partial x} - 4\left(\int\int dx'dy'\; x'Q(x',y',t)\right)\right)Q(x,y,t)\,dW$$



High measurement strength → fast feedback / high energy steady state

Modulating measurement strength

Measure hard (fast control), then weakly (cold steady state)

Restrict to Gaussian states, linear feedback

$$\begin{aligned}
\langle dx_1 \rangle &= \left(2x_3 + 4\alpha x_4^2\right) dt \\
\langle dx_2 \rangle &= \left(-2x_3 + 2k_p x_2 + 4\alpha x_6\right) dt \\
\langle dx_3 \rangle &= \left(x_2 - x_1 + k_p x_3 + 4\alpha x_4 x_6\right) dt \\
\langle dx_4 \rangle &= \left(2x_6 - 4\alpha x_4^2\right) dt \\
\langle dx_5 \rangle &= \left(-2x_6 + \alpha - 4\alpha x_6^2\right) dt \\
\langle dx_6 \rangle &= \left(x_5 - x_4 - 4\alpha x_4 x_6\right) dt
\end{aligned}$$

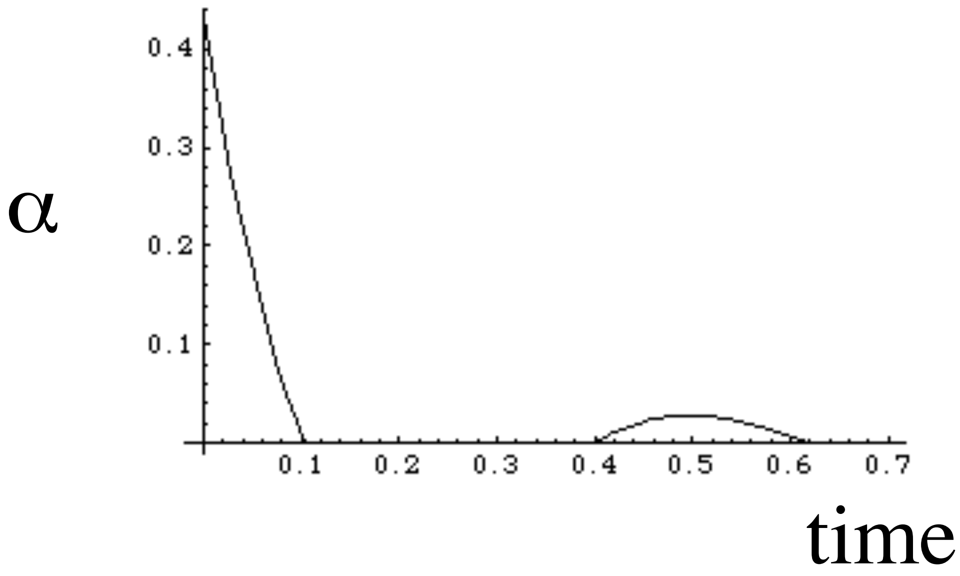
$$\begin{aligned}
x_1 &= q^2 \\
x_2 &= p^2 \\
x_3 &= qp \\
x_4 &= V_q \\
x_5 &= V_p \\
x_6 &= V_{pq}
\end{aligned}$$

What is a good figure of merit?

$$E(t_f) \qquad \int_0^{t_f} E(u)du$$

$$\int_0^{t_f} \left(E(u) + k_c\, \alpha(u)^2\right) du$$

Optimal measurement function



- "Bang-bang" type control

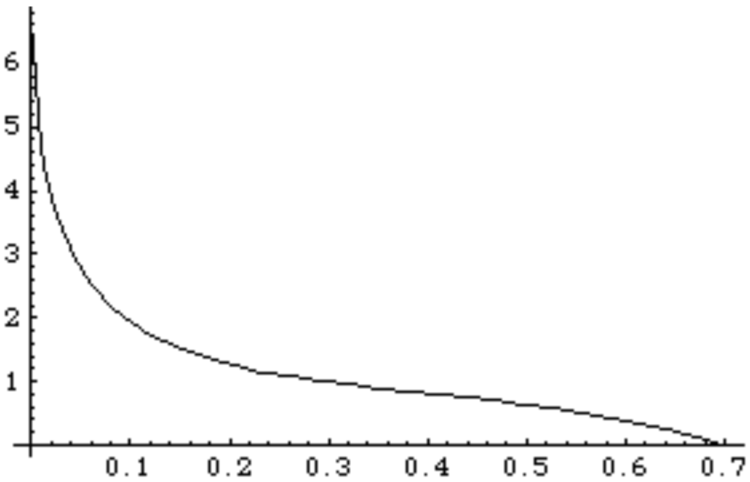
- Hard to compute (particularly as optimal strategy changes)

- Dramatic changes in strategy lead to lack of robustness

Focus on knowledge

Cost $$\int_0^{t_f} (V_q(u)V_p(u) - V_{qp}(u)^2)du$$

α



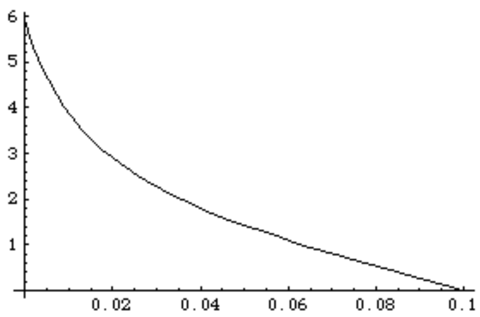
Very stable curve

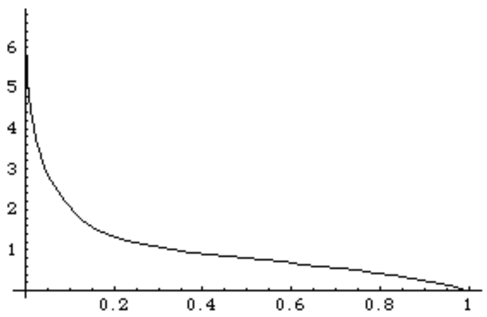
time

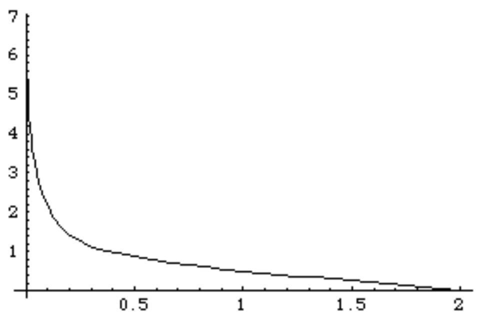
Near optimal energy curve ~1%

Superior final energy! ~10%

α





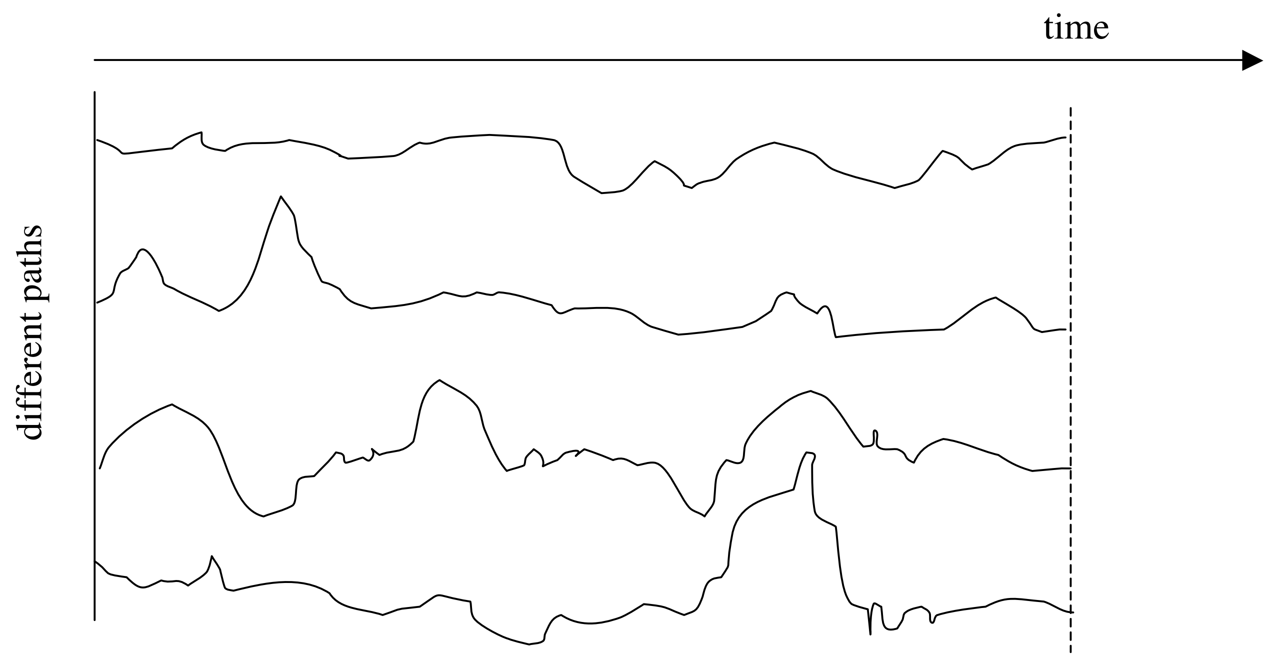


The limitations of brute force

Pumped atom lasers need simultaneous control of spatial mode and statistics

Stochastic FPEs for these systems are easy to generate, but not compute

$$dQ(\alpha,t) = \ldots + \left(\alpha - \int \beta Q(\beta,t)\, d^2\beta\right) Q(\alpha,t)\, dW$$



Summary

1. Including measurement in a simple model of atomic control
   - leads the system to a unique steady state
   - the steady state is above the ground state energy

2. Adjusting the measurement strength can lead to the true ground state efficiently

3. We are developing techniques that allow control theory to be applied to non-analytic, high dimensional models.