Quantum Optics with Cold Atoms and Molecules

- Dissipative Hubbard models

  atoms in lattice + “*phonon*” bath

- [Cold polar molecules]
- [Sub-wavelength / addressable optical lattices]

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SFB
Coherent Control of Quantum Systems

€U networks

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Crispin's
60-th birthday

Innsbruck Oct 2002


unknown theorist

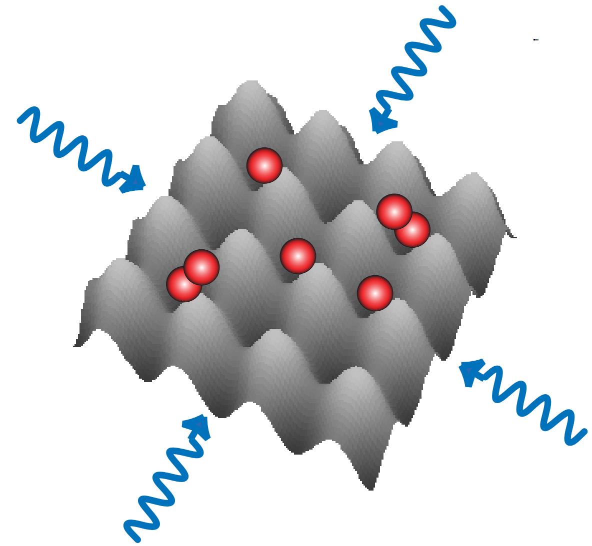
Dissipative dynamics of cold atoms in optical lattices

- quantum optics with cold atoms

Jaksch, ... Gardiner ...
PRL 1998

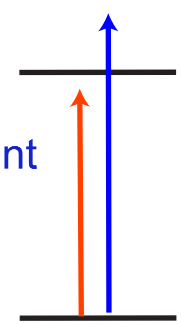
Cold atoms in optical lattices:
1. *Coherent* Hubbard dynamics

- Loading bosonic or fermionic atoms into optical lattices
- Atomic Hubbard models with controllable parameters
  - bose / fermi in 1,2&3D
  - spin models
  - “AMO Hubbard toolbox”

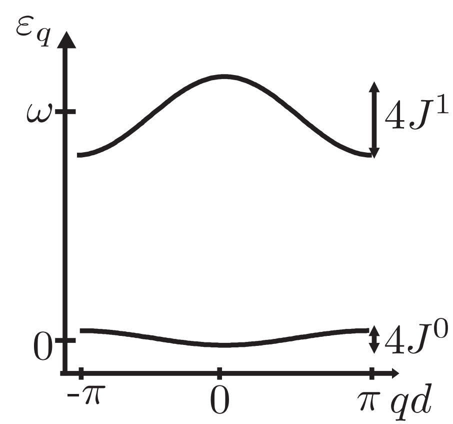


optical lattice as array of microtraps

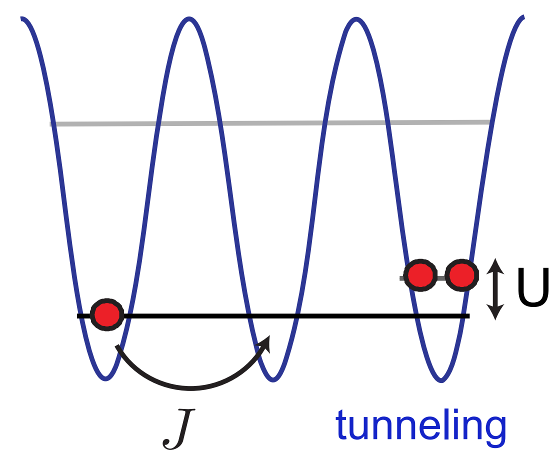
nonresonant laser



AC Stark shift



band structure (1D)



onsite interaction

tunneling

$$\hat{H} = -\sum_{\alpha\neq\beta} J_{\alpha\beta}\hat{a}_\alpha^\dagger a_\beta + \frac{1}{2}U\sum_\alpha \hat{a}_\alpha^\dagger\hat{a}_\alpha^\dagger\hat{a}_\alpha\hat{a}_\alpha$$

single band Hubbard model

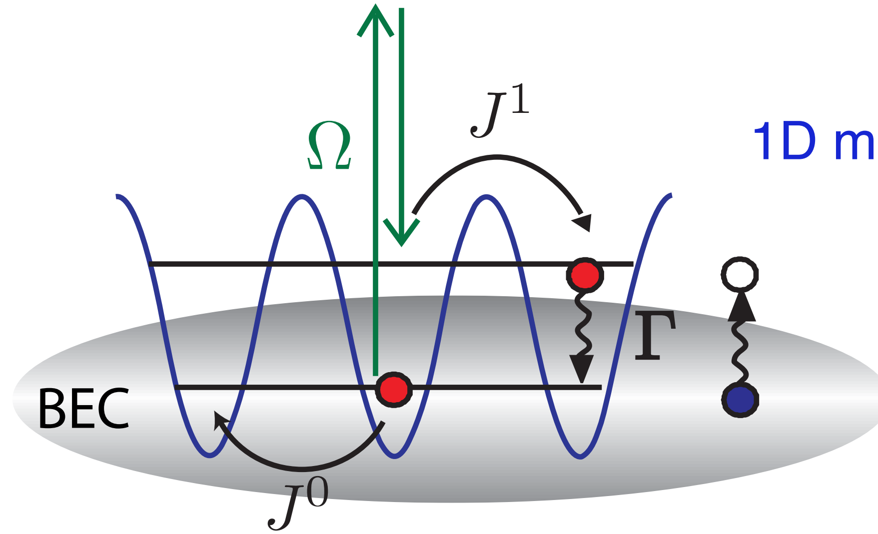
kinetic energy: hopping

interaction: onsite repulsion

“quantum simulators”

2. *Dissipative* Hubbard dynamics

- BEC as a “phonon reservoir”
  - quantum reservoir engineering



1D model

- master equation:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathscr{L}\hat{\rho}$$

- validity (as in quantum optics)
  - ✓ *inter*band transitions
  - ✓ RWA + Born + Markov

- *coherent* Hubbard dynamics

$$H = \ldots$$

✓ two band Hubbard model (1D)
✓ + Raman coupling

- *dissipative* dynamics

$$\mathscr{L}\rho = \sum_k \frac{\Gamma_k}{2}\left(2c_k\hat{\rho}c_k^\dagger - c_k^\dagger c_k\hat{\rho} - \hat{\rho}c_k^\dagger c_k\right)$$

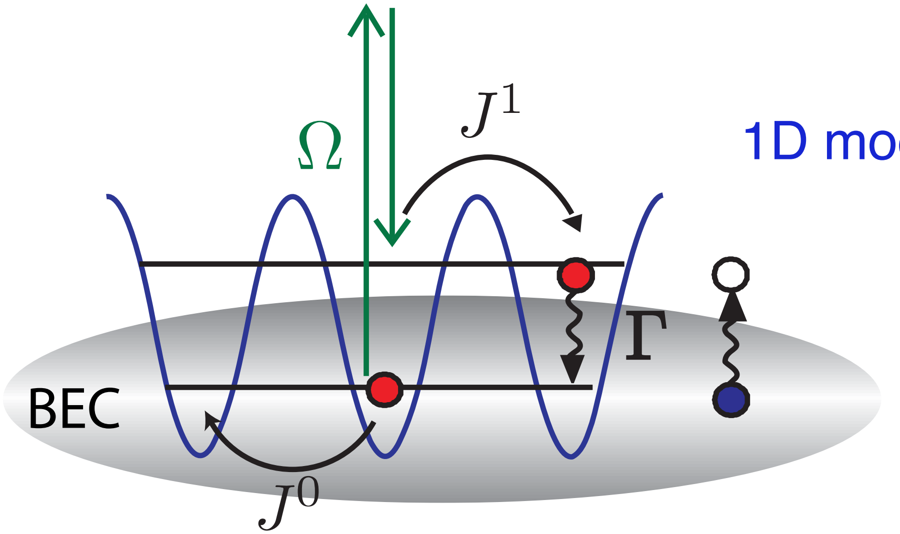
Lindblad form

competing dynamics

2. *Dissipative* Hubbard dynamics

A. Griessner et al. PRL 2006; NJP 2007

- BEC as a "phonon reservoir"
  - quantum reservoir engineering



- master equation:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathscr{L}\hat{\rho}$$

- validity (as in quantum optics)
  - ✓ *inter*band transitions
  - ✓ RWA + Born + Markov

---

as opposed to ...

- Caldeira-Leggett
- linear system-bath couplings, ohmic / superohmic
- quantum phase transitions in Josepshon Junction arrays
- polarons
- phonon mediated interactions

Why (controlled dissipation)?

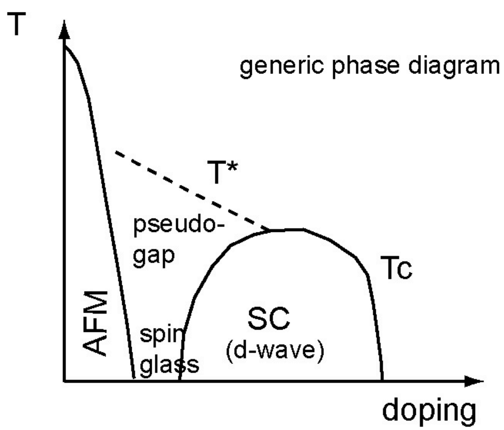
$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}] + \mathscr{L}\hat{\rho}$$

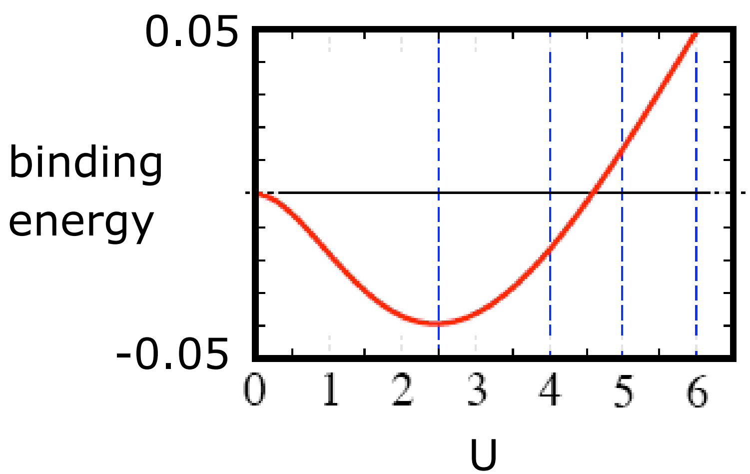
competing dynamics

- why? engineering reservoirs for ...
  - dissipative quantum phase transitions / crossover
  - ...
  - applications: cooling etc.

- Anderson (1987): ground state = resonating valence bond state

high-Tc superconductors





binding energy 4% of width of Bloch band

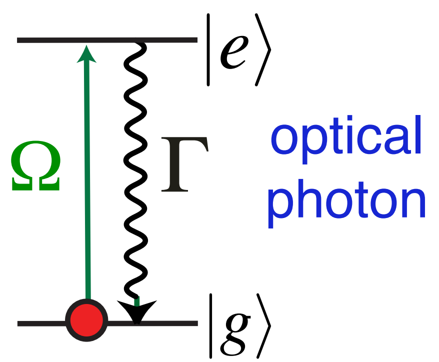
(units of hopping t)

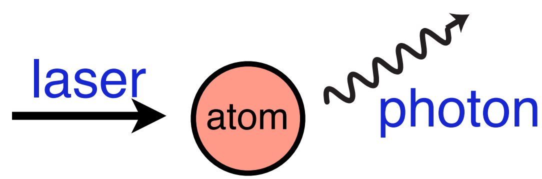
minimal model: two-dimensional one-band Hubbard model

$$\hat{H} = -t \sum_{<ij>,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + h.c. + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$

"think quantum optics"

- driven two-level atom + spontaneous emission





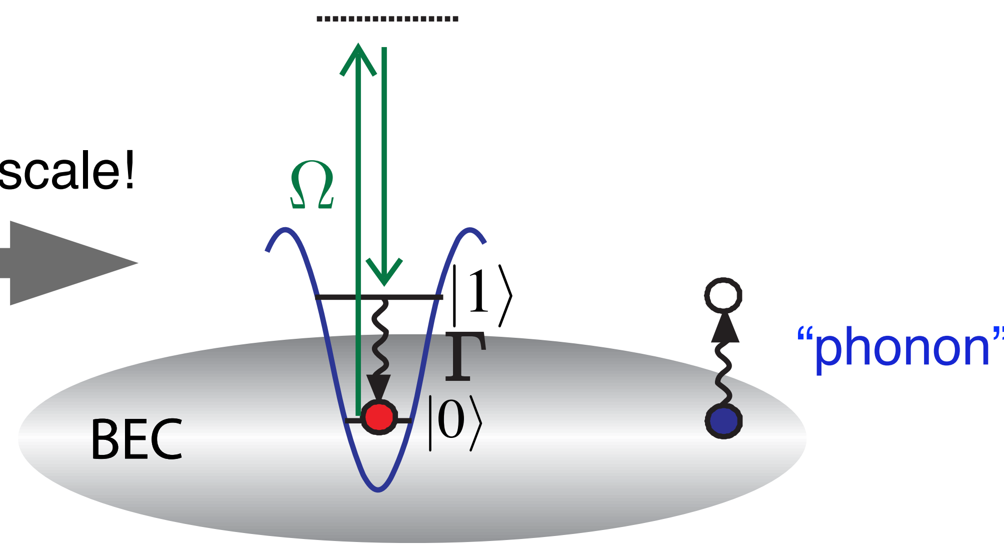
- reservoir: vacuum modes of the radiation field (T=0)

- optical pumping, laser cooling, ...
  - purification of electronic, and motional states

$$\rho_a \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}| \rightarrow |\psi_a\rangle\langle\psi_a| \otimes \rho'$$

energy scale!

- trapped atom in a BEC reservoir



laser assisted atom + BEC collision

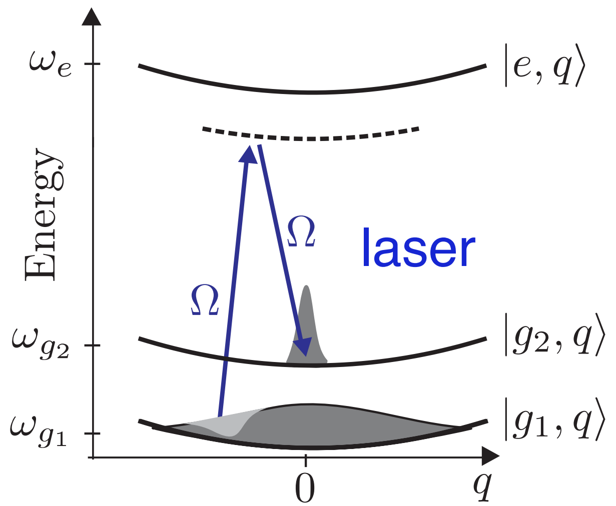
- reservoir: Bogoliubov excitations of the BEC (@ temperature T)

?

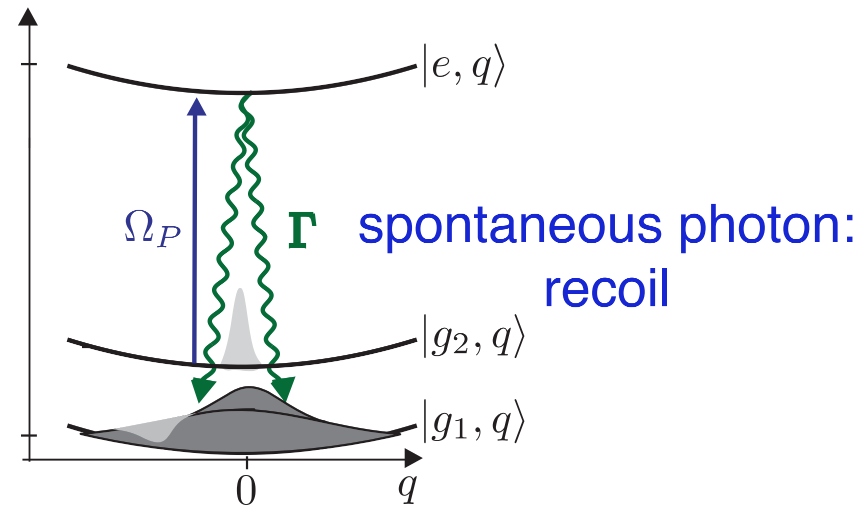
Subrecoil (“dark state”) laser cooling

Raman subrecoil cooling (Kasevich and Chu) (see also: VSCPT Cohen et al.)

step 1: excitation & filtering



step 2: diffusion



- “dark state” laser cooling: accumulate atoms near q≈0
- theory: Levy statistics approach (Cohen et al.)

excitation profile:

$R(q) \sim |q|^{\lambda}$

$\lambda = 2$ square pule

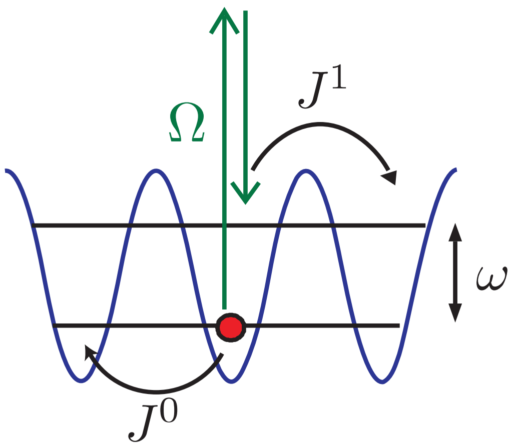
$\lambda = 4$ Blackman pulse

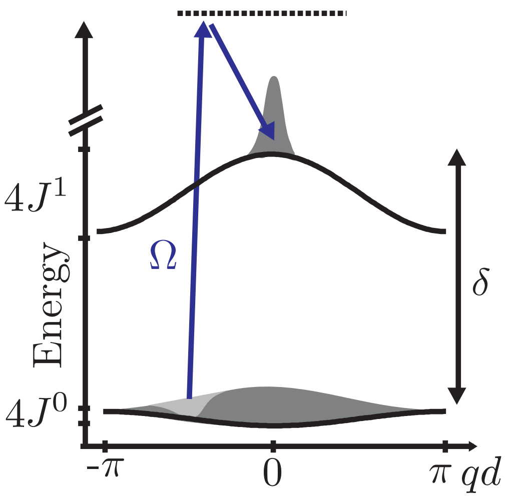
temperature $\frac{1}{2} k_B T = \frac{\delta q^2}{2m} \sim \Theta^{-2/\lambda}$ time

A. Griessner et al. PRL 2006; NJP 2007

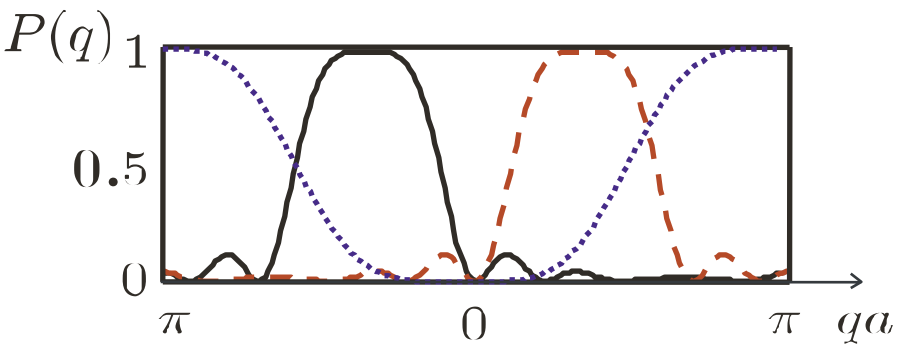
Raman cooling *within* a Bloch band

- step 1: (coherent) quasimomentum selective excitation





Laser: square pulse sequence



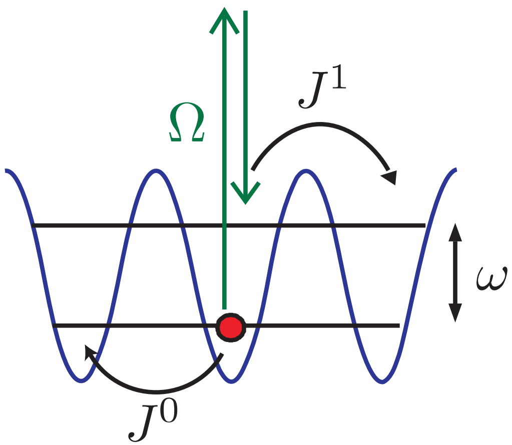
$$P(q) = \frac{\Omega^2}{(\delta_{q+\delta q}^2 + \Omega^2)} \sin^2\left(\sqrt{\delta_{q+\delta q}^2 + \Omega^2}\,\tau/2\right)$$

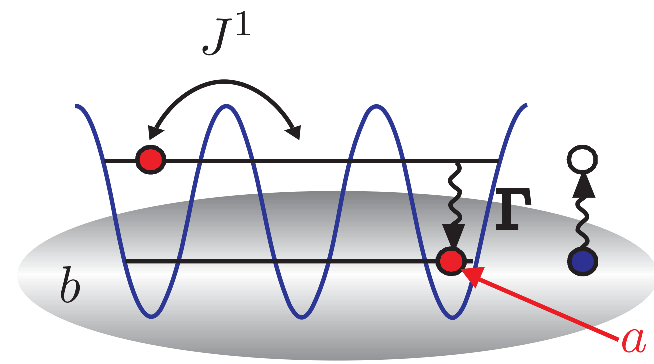
- requirements: $\Omega \ll 8|J^1|$
- Note: relevant energy scale given by $|J^1|$

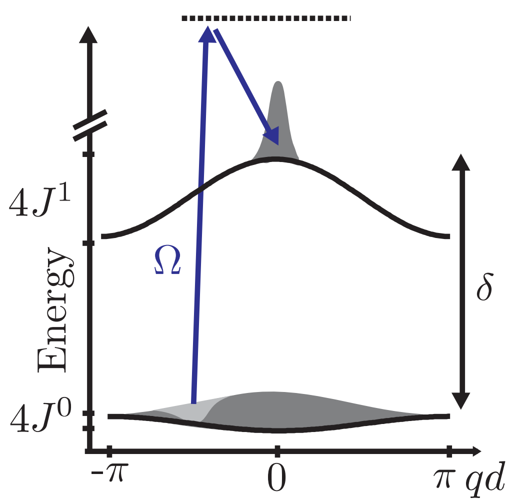
A. Griessner et al. PRL 2006;
NJP 2007

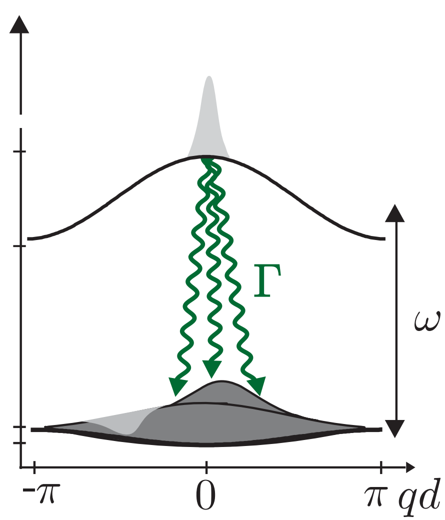
Raman cooling *within* a Bloch band

- step 1: (coherent) quasimomentum selective excitation
- step 2: (dissipative) decay to ground band



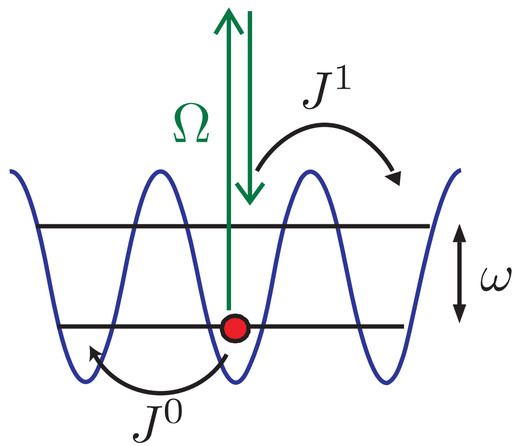


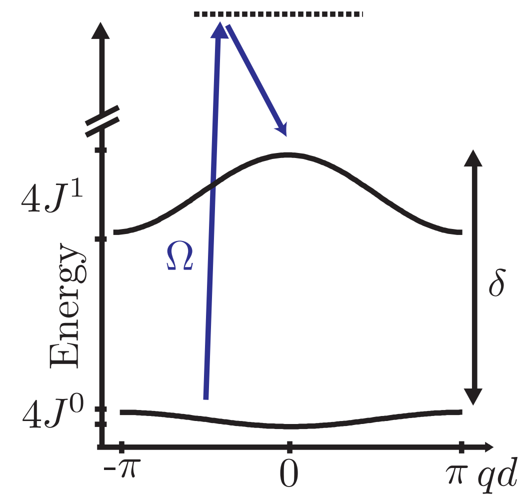




Model: 1. Coherent dynamics

- 1D lattice





Bloch bands

- Hamiltonian

$$\hat{H}_0 = \sum_{q,\alpha} \varepsilon_q^\alpha \left(\hat{A}_q^\alpha\right)^\dagger \hat{A}_q^\alpha + (\omega - \delta) \sum_q \left(\hat{A}_q^1\right)^\dagger \hat{A}_q^1 + \frac{\Omega}{2} \sum_q \left[\left(\hat{A}_q^1\right)^\dagger \hat{A}_{q-\delta q}^0 + \text{h.c.}\right]$$

$$\varepsilon_q^\alpha = -2J^\alpha \cos(qd)$$

Bloch band

Rabi freq.

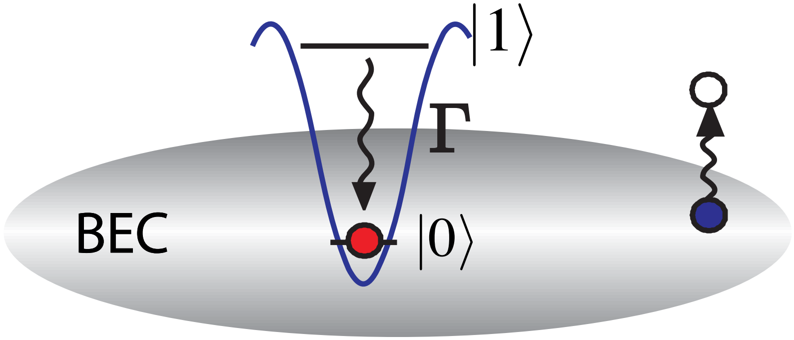
$$\hat{H}_I = \frac{1}{2M} \sum_{q_1,q_2,q_3,\alpha} U^{\alpha\beta} \left(\hat{A}_{q_1}^\beta\right)^\dagger \left(\hat{A}_{q_2}^\alpha\right)^\dagger \hat{A}_{q_3}^\alpha \hat{A}_{q_1+q_2-q_3}^\beta$$

tune via scattering length

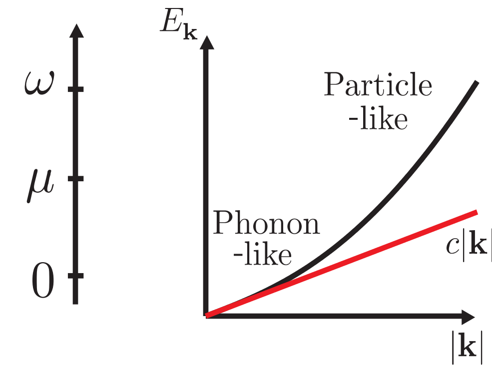
collisional interactions

validity: $J^\alpha, U^{\alpha,\beta'}, \Omega \ll \omega$, $\omega \ll \omega_\perp$

Model: 2. "Spontaneous Emission"



spectrum of Bogoliubov excitations



$$\varepsilon_k = [c^2(\hbar k)^2 + (\hbar k)^4/(2m_b)^2]^{1/2}$$

- BEC reservoir

$$\hat{H}_{\mathrm{BEC}} = E_0 + \sum_{\mathbf{k}\neq 0} \varepsilon(\mathbf{k})\, \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}}$$

$$\hat{\psi}_b = \sqrt{\rho_0} + \delta\hat{\psi}_b$$

$$\delta\hat{\psi}_b = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \left(u_{\mathbf{k}} \hat{b}_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{r}} + v_{\mathbf{k}} \hat{b}^\dagger_{\mathbf{k}} \mathrm{e}^{-\mathrm{i}\mathbf{k}.\mathbf{r}} \right)$$

Bogoliubov

- interaction: *inter*band 0 - 1

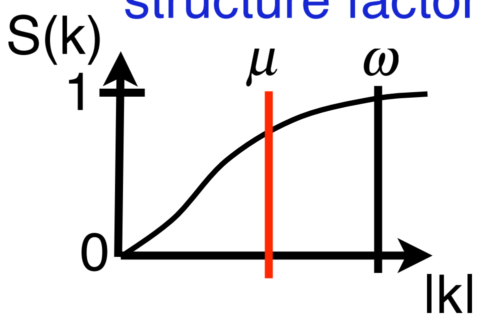
$$\hat{H}_{\mathrm{int}} = g_{ab} \int \hat{\psi}^\dagger_a(\mathbf{r}) \hat{\psi}_a(\mathbf{r}) \hat{\psi}^\dagger_b(\mathbf{r}) \hat{\psi}_b(\mathbf{r}) \mathrm{d}^3\mathbf{r}$$

$$\sim g_{ab} \sum_{\mathbf{k}} S(\mathbf{k},\omega)^{1/2} \langle w_1 | \mathrm{e}^{\mathrm{i}\mathbf{k}.\mathbf{r}} | w_0 \rangle \hat{b}_{\mathbf{k}} |1\rangle\langle 0| + \mathrm{h.c.}$$

"spontaneous emission"

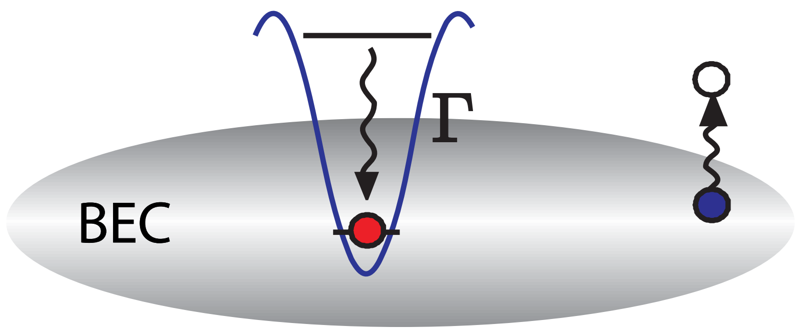
≈ 1

structure factor

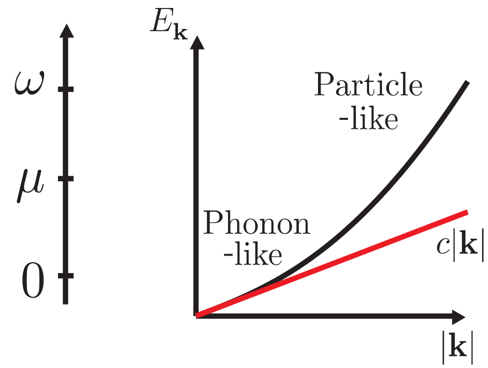


$$S(\mathbf{k}) = (u_{\mathbf{k}} + v_{\mathbf{k}})^2 = \frac{|\mathbf{k}|^2}{2m_b E_{\mathbf{k}}}$$

"Spontaneous Emission"



spectrum of Bogoliubov excitations



- interband transitions spontaneous emission rate
  - typical numbers

$$\Gamma = 2\pi \times 1.1 \text{ KHz}$$

weak coupling

$$a_s = 100 a_0$$ scattering length

$$\rho_0 = 5 \times 10^{14} \text{cm}^{-3}$$ density

$$\omega = 2\pi \times 100 \text{ KHz}$$ trap frequency

  - tunability

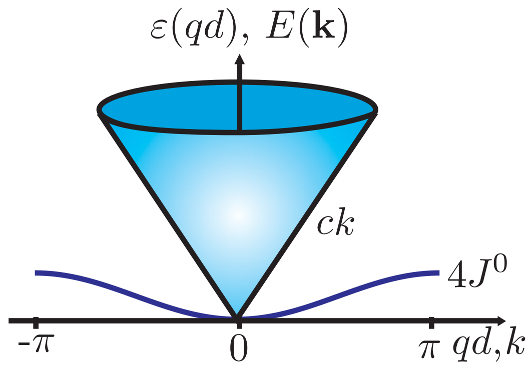
$$\Gamma \sim \rho_0 a_s^2 \sqrt{\omega}$$

scattering length:
magnetic or optical Feshbach resonance

density

- interaction: *intra*band ...

$$\varepsilon^0_{q\approx 0} = \varepsilon^0_{q'} + c|\mathbf{k}|$$
$$q = q' + k$$



forbidden if $J^0 < \dfrac{\sqrt{\mu\omega_R m_a/(2m_b)}}{\pi}$

- ✓ no heating / cooling due to intraband transitions
- ✓ we ignore intraband processes in the following
- ✓ Rem.: validity of master equation ...

We can cool to temperatures lower than the BEC

Master equation

- ... in analogy with spontaneous emission ($k_B T \ll \hbar\omega$, i.e. $T = 0$)

$$\mathscr{L}\hat{\rho} = \sum_k \frac{\Gamma_k}{2}\left(2c_k\hat{\rho}c_k^\dagger - c_k^\dagger c_k\hat{\rho} - \hat{\rho}c_k^\dagger c_k\right)$$

1D momentum along lattice axis

$$|k| \le k_{\max} = \sqrt{2m_b\omega}$$

energy conservation

quantum jump operator

$$c_k \equiv \sum_j (\hat{a}_j^0)^\dagger(\hat{a}_j^1)\mathrm{e}^{-ikx_j} = \sum_q (\hat{A}_{q-k}^0)^\dagger \hat{A}_q^1$$

modulo first Brillouin zone

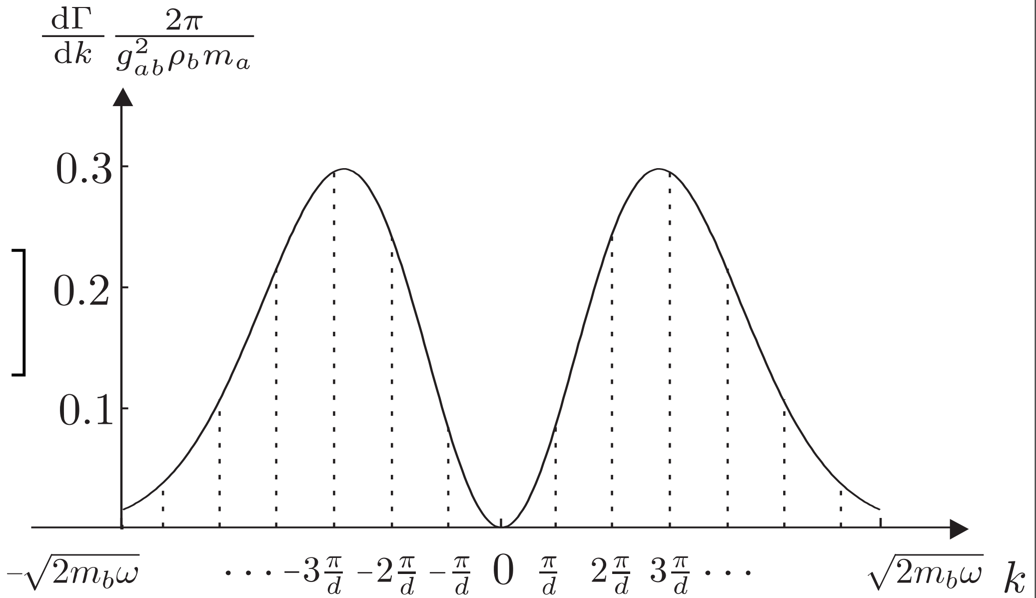
- spontaneous emission rate $\Gamma = \sum_k \Gamma_k$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}k} \hat{=} \frac{L}{2\pi}\Gamma_k = \frac{g_{ab}^2\rho_b m_a a_0^2 k^2}{4\pi}\mathrm{e}^{-a_0^2k^2/2}$$

$$\Gamma = \frac{g_{ab}^2\rho_b m_b}{2\pi a_0}\left[\sqrt{2\frac{m_b}{m_a}}\mathrm{e}^{-\frac{m_b}{m_a}} - \sqrt{\frac{\pi}{2}}\mathrm{erf}\left(\sqrt{\frac{m_b}{m_a}}\right)\right]$$

(1) $k_{\max} \gg \pi/d$, no superradiance

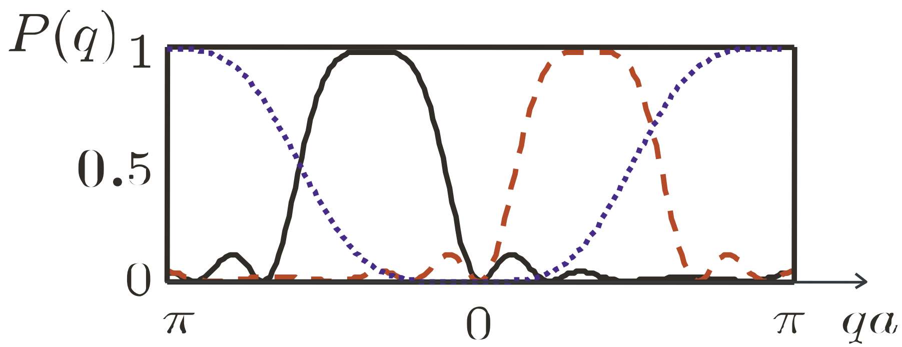
(2) $k_{\max} < \pi/d$. [superradiance]

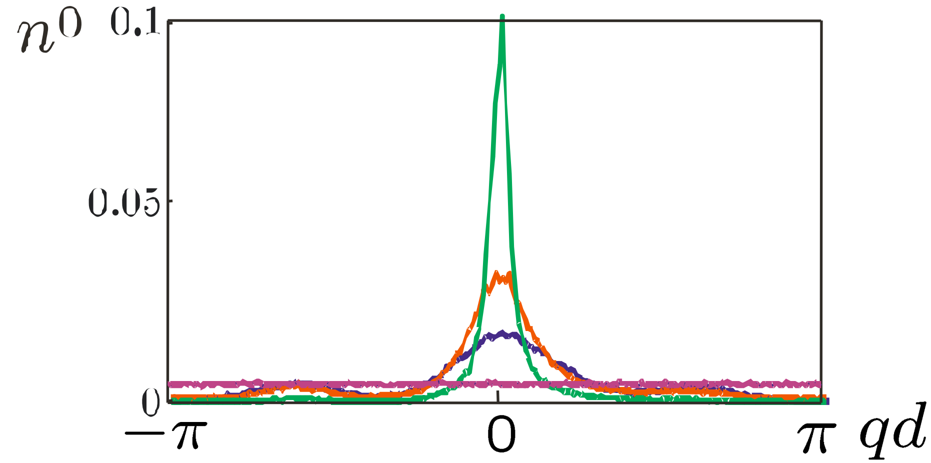


Results: single atoms

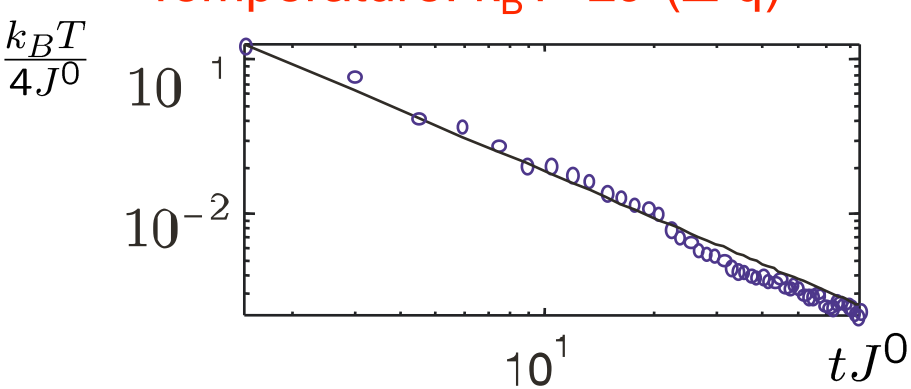
- Ground state q=0 momentum peak $4J^0 \ll k_B T \ll \omega$.
- Quantum trajectory simulation of the master equation

Laser: square pulse sequence

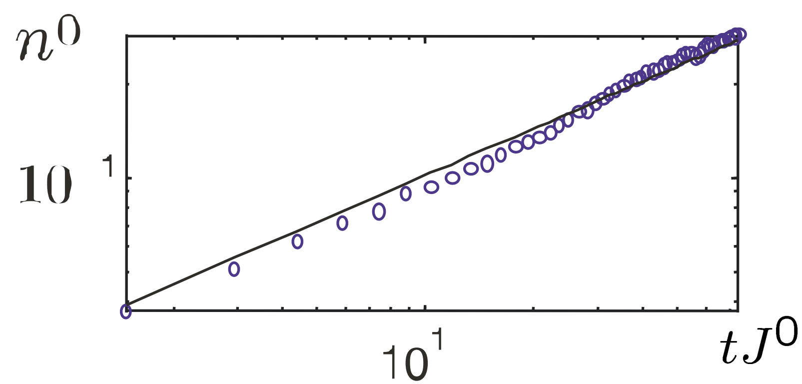




Temperature: $k_B T = 2J^0(\Delta q)^2$



Dark state occupation: $n^0(q=0)$



- Typical temperatures $k_B T/4J^0 \sim 2 \times 10^{-3}$ in $t_f J^0 \sim 50$
- Analysis in terms of Levy flights

Many (non-interacting) bosons

- Assume: we can switch off interaction between bosons $a_{aa} \to 0$ with Feshbach resonance; independent bosons
- Ground state cooling: $q = 0$ peak in momentum distribution
- Numerical analysis: Quantum Boltzmann master equation

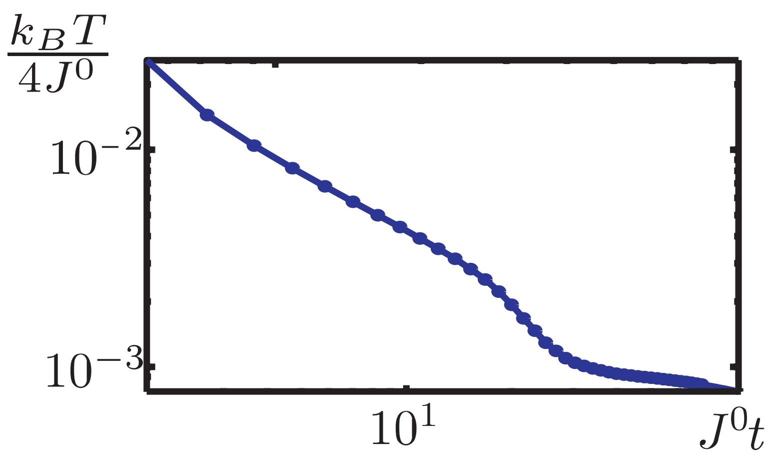
$$\dot{w}_{\mathbf{m}} = \sum_{k,q} \Gamma_k \left[\mathrm{m}^0_{q-k}(1 \pm \mathrm{m}^1_q) w_{\mathbf{m}'} - \mathrm{m}^1_q (1 \pm \mathrm{m}^0_{q-k}) w_{\mathbf{m}} \right]$$

↑ occupation of momentum state q in Bloch band

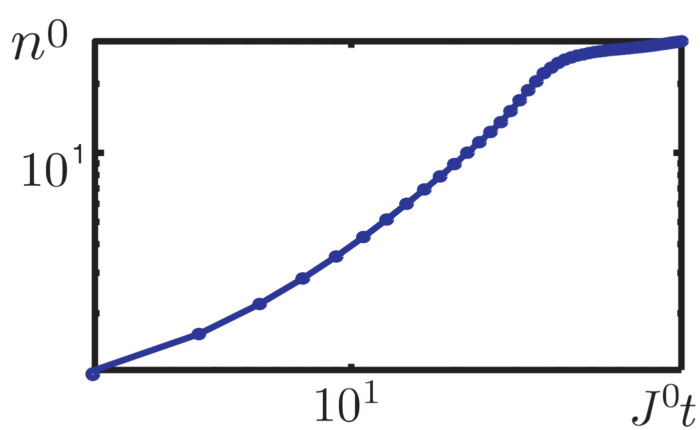
We failed to apply DMRG type ideas because our temperatures are too low ☹

QBME is a rate equation for $w_{\mathbf{m}} \equiv \langle \mathbf{m} | \rho | \mathbf{m} \rangle$, i.e. classical configurations $w_{\mathbf{m}}$ of atoms occupying momentum states $\mathbf{m} = [\{m^0_q\}_q, \{m^1_q\}_q]$ in the two Bloch bands.

Temperature:



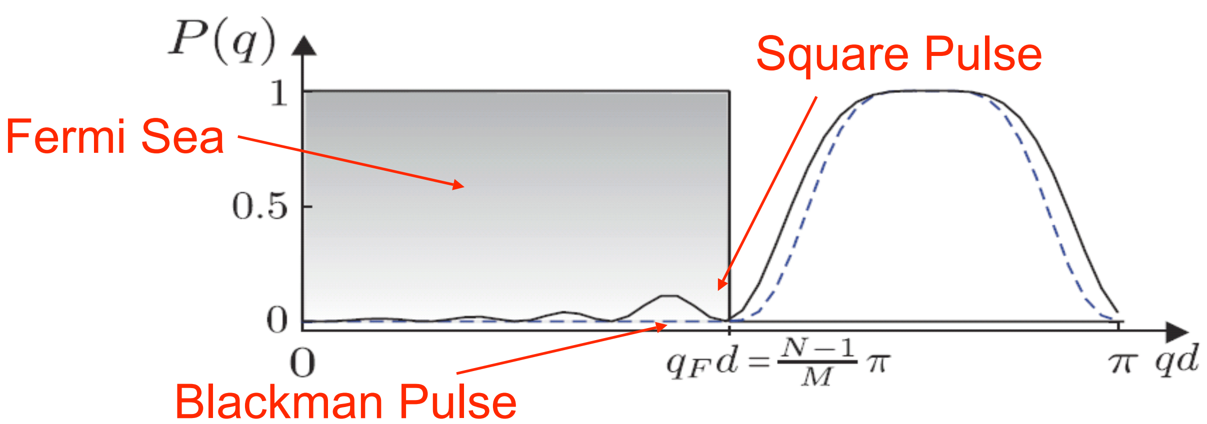
Dark state occupation: n^0(|qd|<0.06)

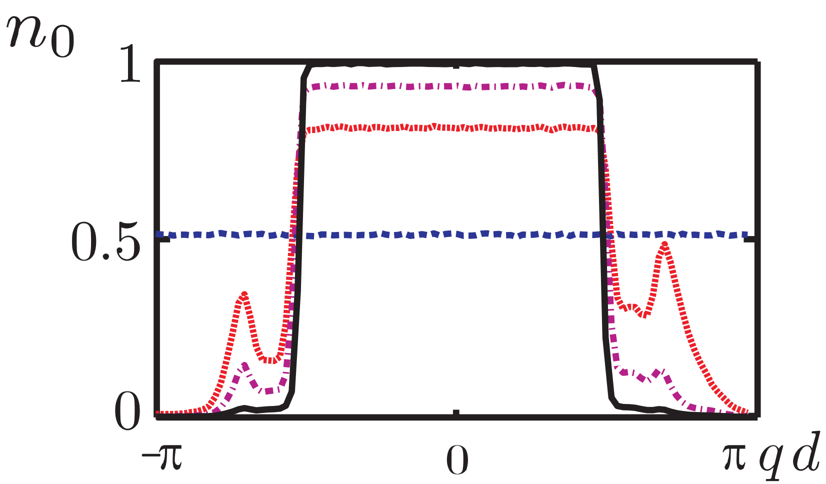


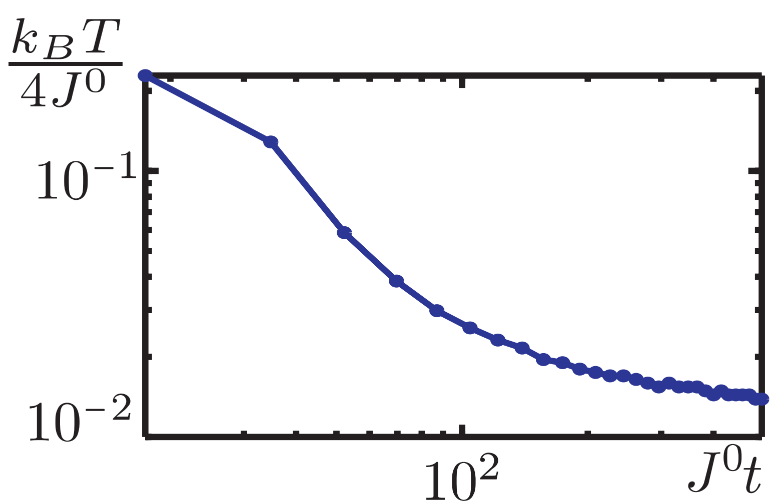
- Bosonic enhancement of cooling

Many fermions

- Many spin-polarized (non-interacting) fermions
- Ground state: filled Fermi sea







- Typical temperatures $k_B T/4J^0 \sim 10^{-2}$ in $t_f J^0 \sim 500$
- Slowing down due to Pauli blocking

Cooling for *strongly correlated* many body systems?

- Strategy (within our model):
  - step 1: perform cooling in absence of interactions (Feshbach)
  - step 2: adiabatically ramp up interactions
  - ... seems to work well for examples studied (1D: DMRG a la Vidal ...)

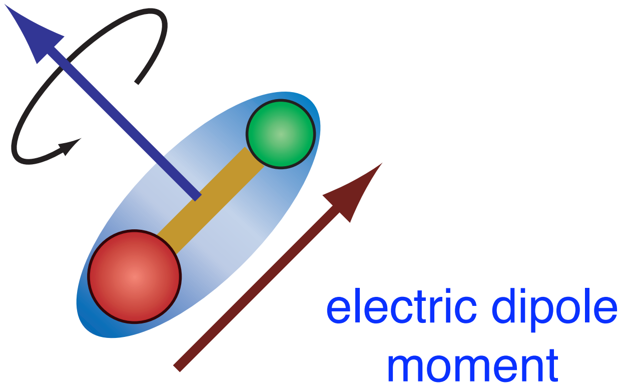
- Question (within our model): cooling in the presence of interactions?
  - answer: ☹

A. Griessner, A. J. Daley, S. R. Clark, D. Jaksch, and P. Zoller,
Dark state cooling of atoms by superfluid immersion.
Phys. Rev. Lett **97**, 220403 (2006).

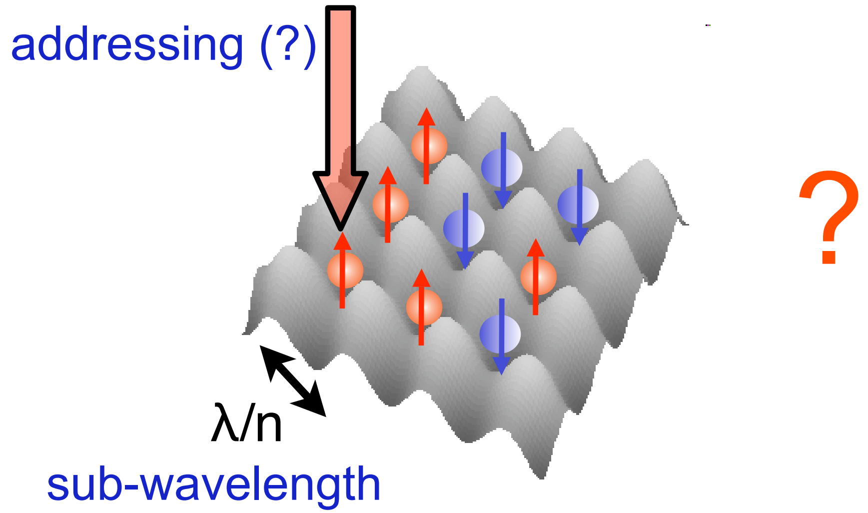
A. Griessner, A. J. Daley, S. R. Clark, D. Jaksch, and P. Zoller,
Dissipative dynamics of atomic Hubbard models coupled to a phonon bath:
Dark state cooling of atoms within a Bloch band of an optical lattice.
New Journal of Physics, in print (2007)

... other recent work

- **Cond mat & quantum information with cold polar molecules**
  - what's new? ... electric dipole moment
    - couple rotation to DC / AC microwave fields
    - strong dipole-dipole / long range couplings

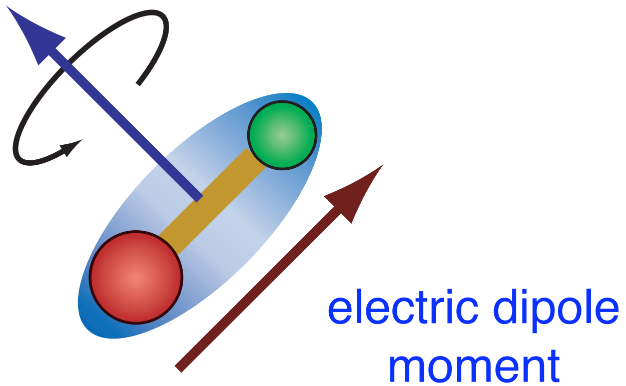


- **Addressable sub-wavelength optical lattices**



Cold polar molecules

- what's new? ... electric dipole moment
  - couple rotation to DC / AC microwave fields
  - strong dipole-dipole / long range couplings

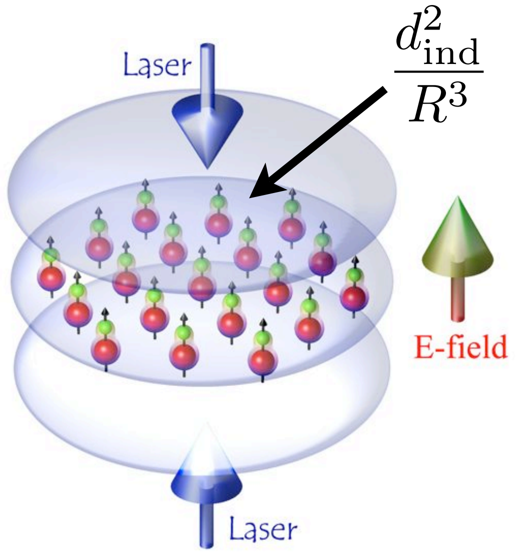


Condensed matter aspects

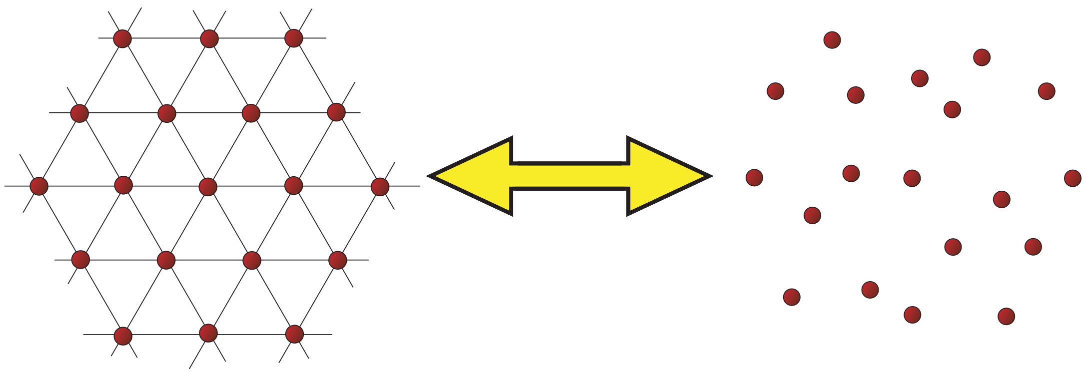
- Self-assembled “dipolar crystals” with cold polar molecules

dipolar crystal:

induced &
aligned dipoles



Quantum melting
- appearance of a crystalline phase
- quantum melting to a superfluid phase



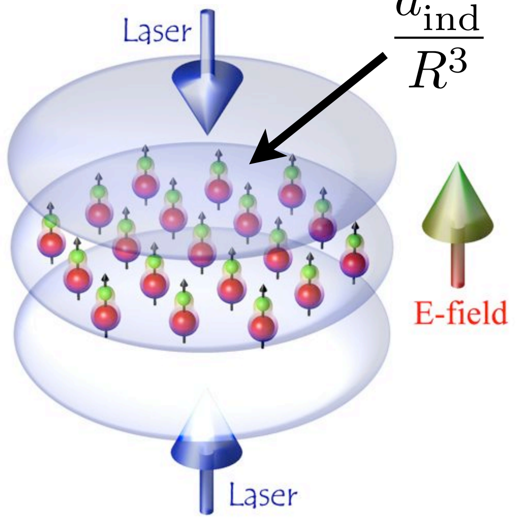
H.P.Büchler, E.Demler, M.Lukin, A. Micheli, N.V.Prokof'ev, G.Pupillo, PZ, PRL (2007)

- Self-assembled “dipolar crystals” with cold polar molecules

dipolar crystal:

induced &
aligned dipoles

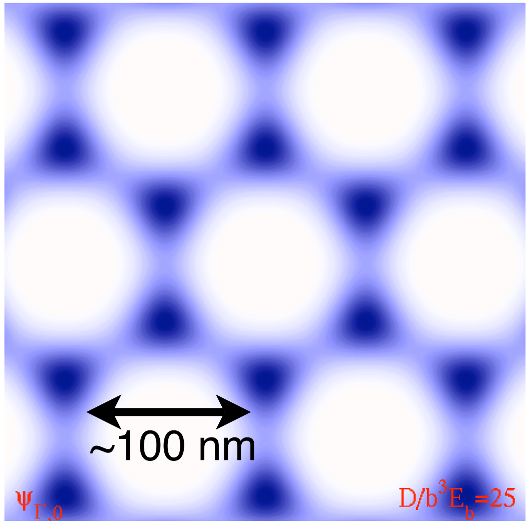
$$\frac{d_{\mathrm{ind}}^2}{R^3}$$



H.P.Büchler, E.Demler, M.Lukin, A. Micheli, N.V.Prokof'ev, G.Pupillo, PZ, PRL (2007)

applications:

atoms in dipolar lattices:
Hubbard models + phonons



G. Pupillo, M. Ortner et al., work in progress

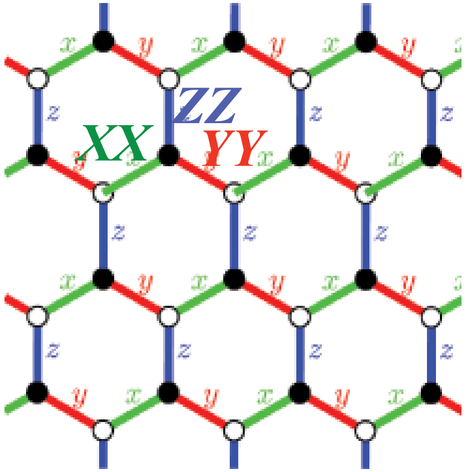
quantum information:
- memory
- ion-trap type quantum computing

Condensed matter aspects

- Spin toolbox with cold molecules in optical lattices

$$H_{\text{spin}} = J_\perp \sum_{\text{x-lks}} \sigma_x^i \sigma_x^j + J_\perp \sum_{\text{y-lks}} \sigma_y^i \sigma_y^j + J_z \sum_{\text{z-lks}} \sigma_z^i \sigma_z^j$$

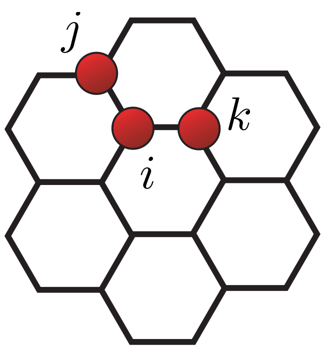
Kitaev model



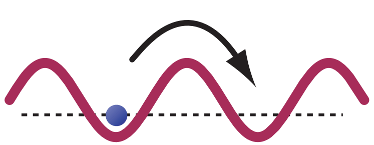
A. Micheli, G. Brennen, PZ, Nature Physics 2006

- Extended Hubbard models in 1D and 2D in optical lattices

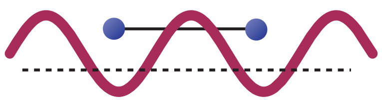
$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} \sum_{i \neq j} \cancel{U_{ij}} n_i n_j + \frac{1}{6!} \sum_{i \neq j \neq k} W_{ijk} n_i n_j n_k .$$



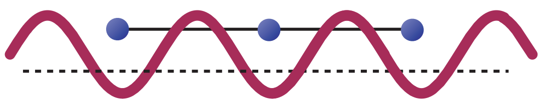
hopping



tunable two-body interaction



strong repulsive off-site three-body interaction



compare: string net Fidkowski et al., cond-mat/0610583

H.P. Büchler, A. Micheli, PZ, preprint

1D hard core Boson with three-body

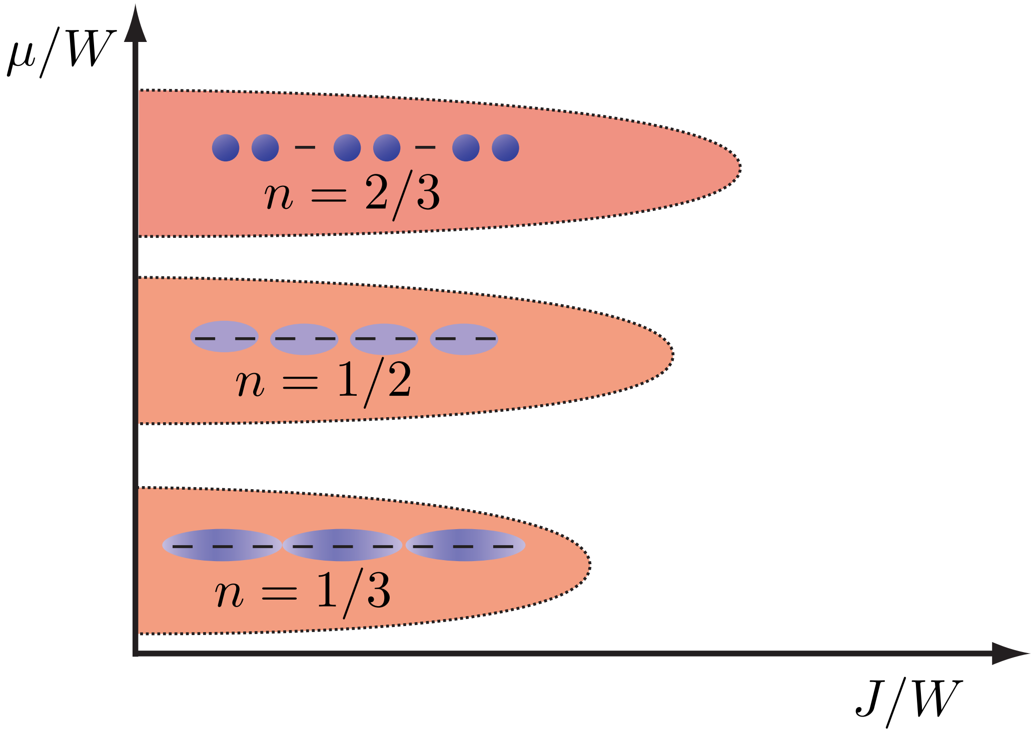
$$H = -J \sum_i \left[b_i^\dagger b_{i+1} + b_{i+1}^\dagger b_i \right] + W \sum_i n_{i-1} n_i n_{i+1}$$

Bosonization

- hard-core bosons
- instabilities for densities:

$$n = 2/3 \quad n = 1/2 \quad n = 1/3$$

- quantum Monte Carlo simulations (in progress)



Critical phase

- algebraic correlations
- compressible
- repulsive fermions

Solid phases

- excitation gap
- incompressible
- density-density correlations

$$\langle \Delta n_i \Delta n_j \rangle$$

- hopping correlations (1D VBS)

$$\langle b_i^\dagger b_{i+1} b_j^\dagger b_{j+1} \rangle$$