Thermalisation and vortex formation in a mechanically perturbed condensate

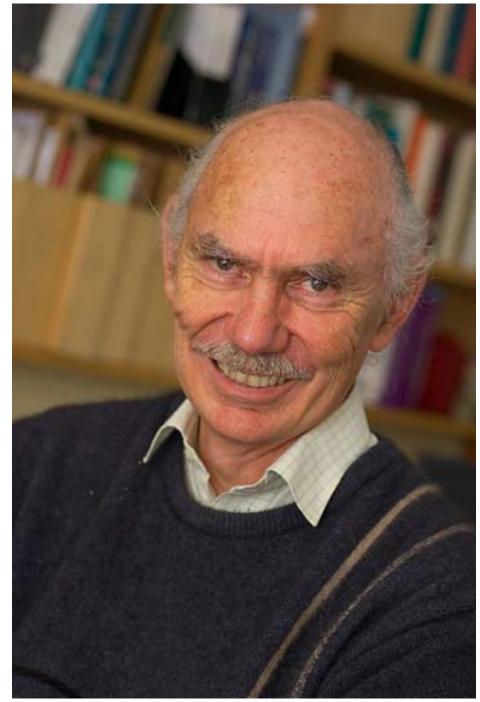
R.J. Ballagh and Tod Wright

Jack Dodd centre for Photonics
and Ultra-cold Atoms

Department of Physics
University of Otago
New Zealand

ULTRA-COLD ATOMS

UNIVERSITY of OTAGO



Jack Dodd Centre

Blair Blakie
Crispin Gardiner
Tod Wright

ACQAO

Ashton Bradley
Matthew Davis

This Talk

Dynamics of finite temperature condensates

1] Vortex lattice via stirring a condensate

- classical field formalism
- formation and role of thermal cloud

2] BKT transition in an optical lattice

- recent experiment of Cornell's group
- preliminary dynamical description

Vortex Lattice Formation

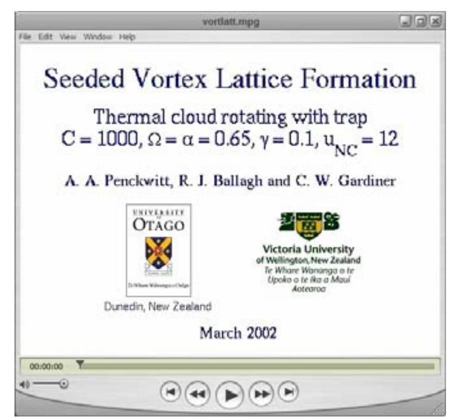
(I) by condensation

Experiment of Cornell's group (evaporatively cool a rotating thermal cloud)

- Cannot simulate formation with unitary equation (e.g. GPE)
- Need dissipation
- Thermal cloud plays a key role

'Fudge' Equation

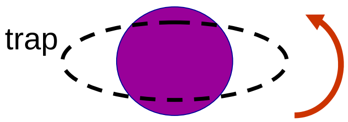
$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \left(H_0 + u|\psi|^2\right)\psi(\mathbf{x},t) + i\gamma\left(\mu_{NC} + \boldsymbol{\alpha}\cdot\mathbf{L} - i\hbar\tfrac{\partial}{\partial t}\right)\psi(\mathbf{r},t)$$

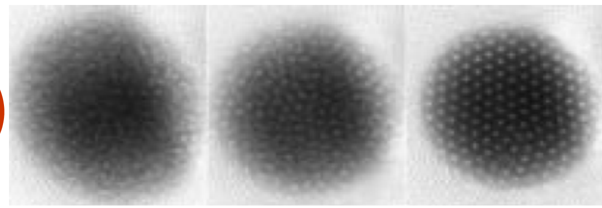


Vortex Lattice Formation

(II) by stirring a condensate

Key experiments: *Ketterle MIT, Dalibard ENS,*

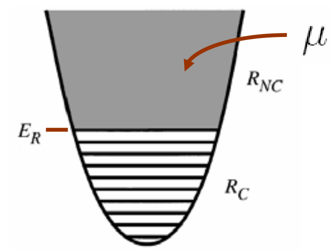




50ms 150ms 300ms

- Creation of thermal cloud by stirring
- *Various theoretical models .....*
- Wish to understand and model this mechanism

The 'Classical Field' method



Formal development

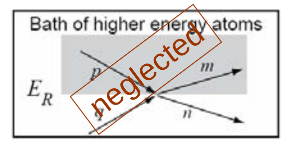
Master equation (eliminate R_{NC})

Wigner function

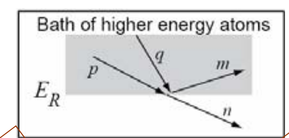
Stochastic PDE for stochastic field $\alpha(\mathbf{x},t)$

Interactions with the bath

'scattering' terms



'growth terms'



$$d\alpha(\mathbf{x},t) = -\frac{i}{\hbar}\mathcal{P}L_{GP}\alpha(\mathbf{x},t)dt + \underbrace{\gamma(\mu - L_{GP})\,\alpha(\mathbf{x},t)dt}_{\text{neglected}} + \underbrace{dW_G(\mathbf{x},t)}_{\text{neglected}}$$

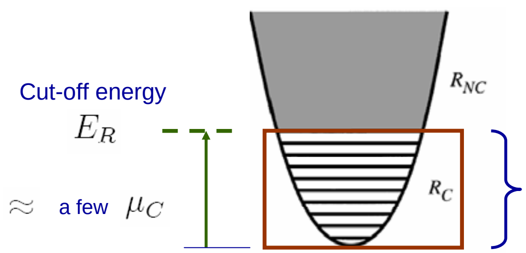
$$\alpha(\mathbf{x},t) \rightarrow \psi(\mathbf{x},t)$$

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \mathcal{P}L_{GP}\psi(\mathbf{x},t)$$

- stochastic initial condition (vacuum and thermal noise)

$$L_{GP}\psi(\mathbf{x},t) \equiv \left[H + u|\psi(\mathbf{x},t)|^2\right]\psi(\mathbf{x},t)$$

Field is **restricted** to **condensate band** $\psi(\mathbf{x},t) = \sum_{n\in R_C}\alpha_n(t)Y_n(\mathbf{x})$ $\langle\mathbf{x}|n\rangle$



Projector enforces this restricted evolution

$$\mathcal{P} = \sum_{n\in R_C}|n\rangle\langle n|$$

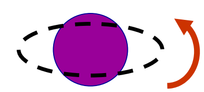
i.e. we are examining dynamics of thermalisation in the condensate band

Region of Validity in Practice

- Sufficiently high *particle density in position space*.
- Initially most modes *unoccupied* and some modes *highly occupied*

Solve in 2D in rotating frame

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \mathcal{P}L_{GP}\psi(\mathbf{x},t)$$



Ω

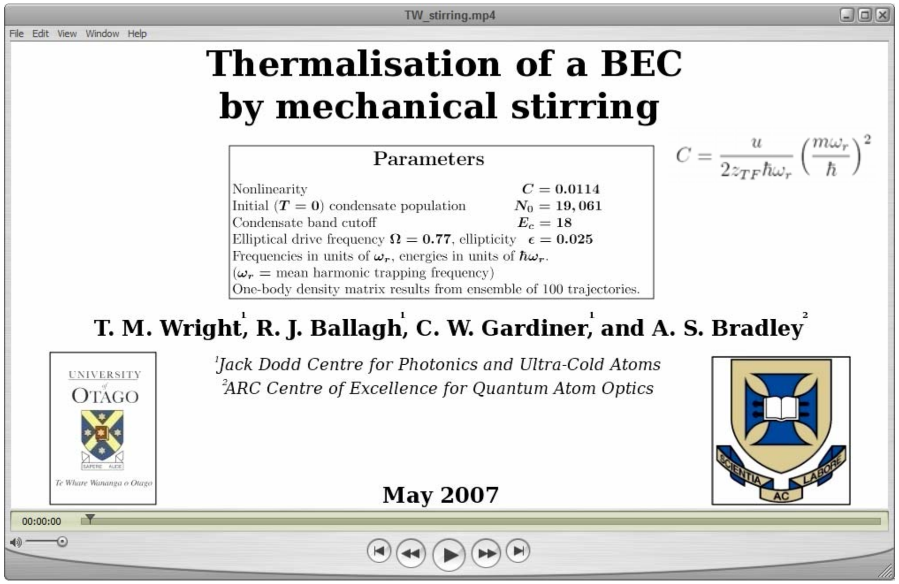
$$L_{GP}\psi(\mathbf{x},t) \equiv \left[-\frac{\hbar^2\nabla^2}{2m} + i\hbar\Omega\cdot(\mathbf{x}\times\nabla) + V(\mathbf{x}) + u|\psi(\mathbf{x},t)|^2\right]\psi(\mathbf{x},t)$$

Initial state *T* = 0 condensate + vacuum noise

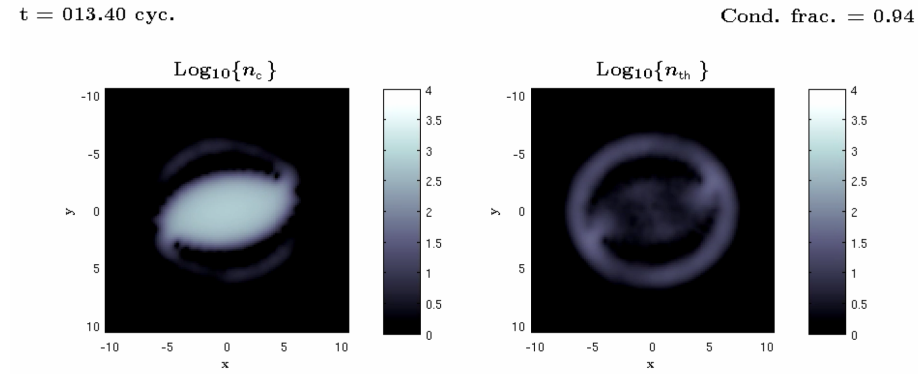
$\mu_C = 8.4\hbar\omega_r$

Basis modes $Y_n(\mathbf{x})$: Gauss-Laguerre

cutoff at $E_R = 18\hbar\omega_r$



Ensemble of 100 simulations



Condensate

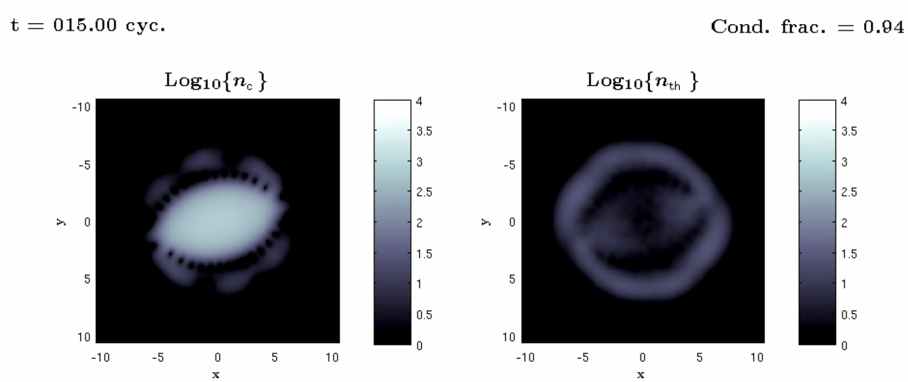
Thermal cloud

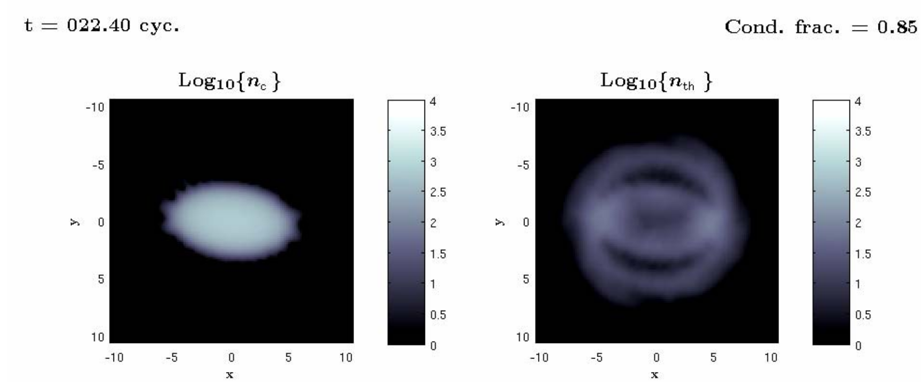
From one-body density matrix

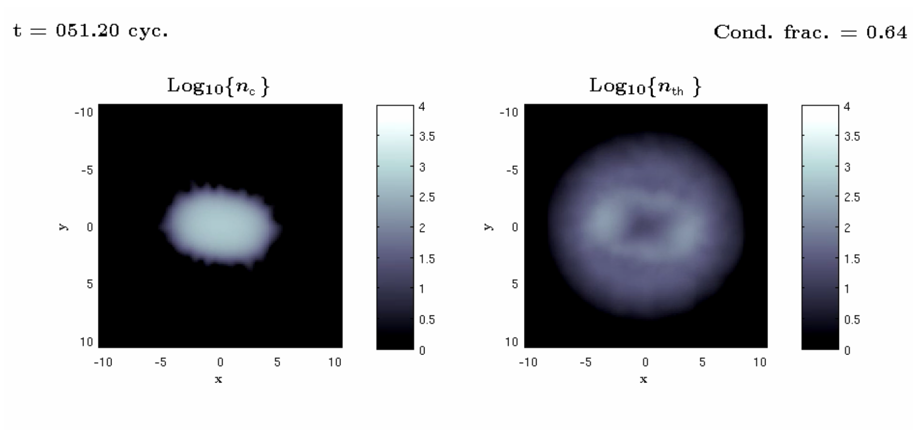
(everything else)

$$\langle \rho(\mathbf{x}, \mathbf{x}', t)\rangle = \langle \psi^*(\mathbf{x}, t)\psi(\mathbf{x}', t)\rangle$$

largest eigenvalue (and eigenvector)

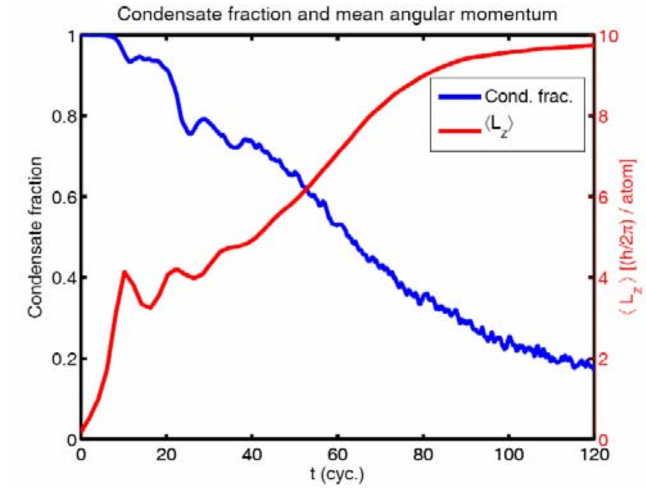




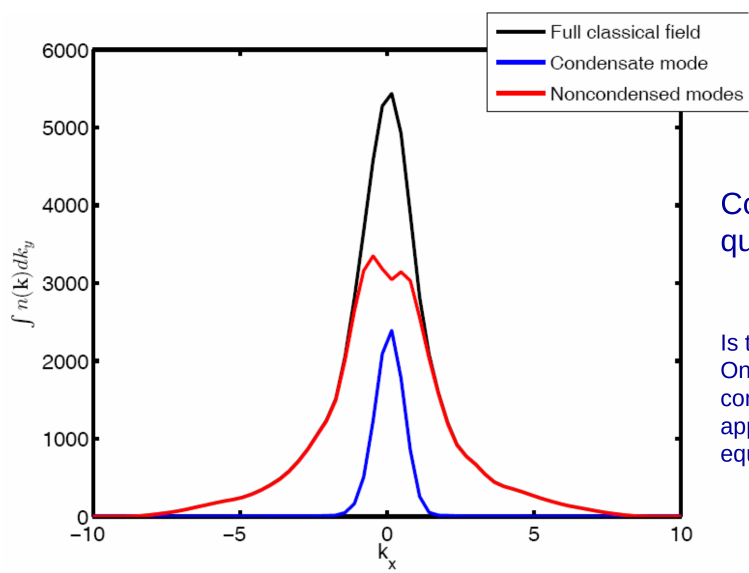


Evolution of

- condensate fraction
- mean angular momentum



Momentum Distribution

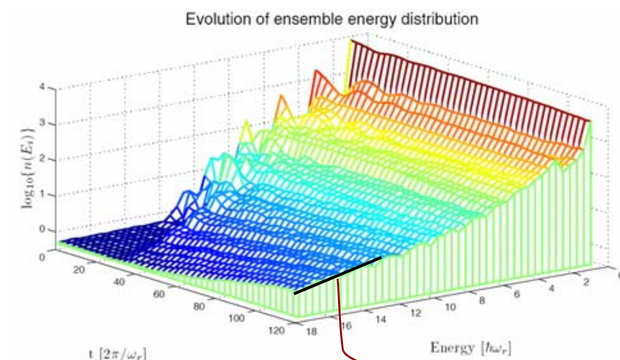


Conceptual question arises:

Is the Penrose-Onsager definition of condensed fraction appropriate for non-equilibrium ?

Extracting thermodynamic quantities of the thermal field

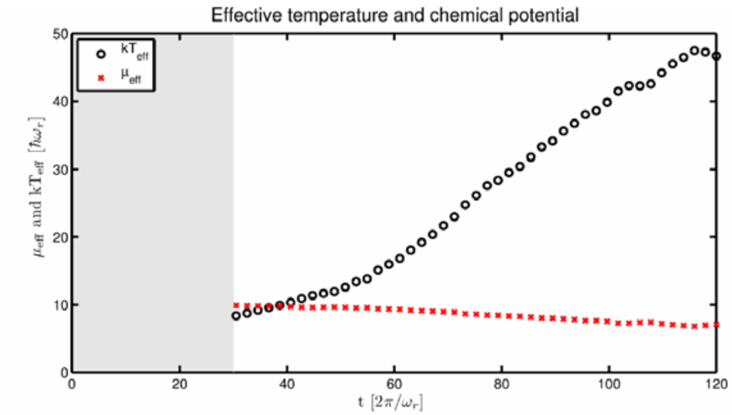
Energy distribution over non-interacting modes



Fit Bose distribution

Have neglected meanfield shift of mode energies

Evolution of μ_{th} and T



- Substantial thermalisation of field in 2D
- May require 3D treatment to describe relaxation to a rigid vortex lattice

Part Two

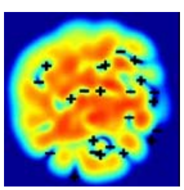
BKT transition in an optical lattice

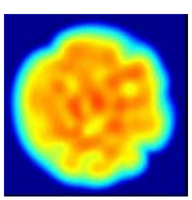
Berezinskii, Kosterlitz and Thouless (BKT) transition

- is a 2D phenomenon
- vortices are central objects

Simula & Blakie

PRL 96,020404, 2006





Temperature

Thermal

free vortices

T_{BKT}

BKT

'bound' vortex pairs

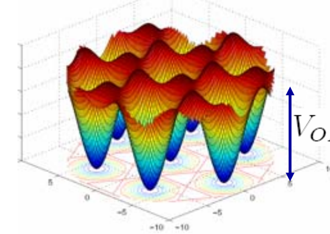
T_c

BEC

Influence of harmonic trap

BKT In a lattice of 2D Bose condensates

e.g. Optical Lattice



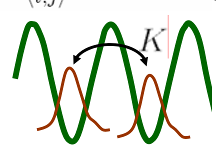
Previous treatment

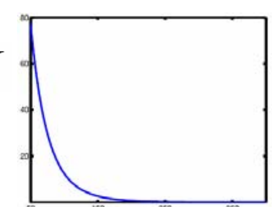
- based on **Bose-Hubbard** model
- tight binding, localised mode expansion

$$\hat{\psi}(\mathbf{x}) = \sum \hat{b}_i w(\mathbf{x} - \mathbf{x}_i)$$

$$\hat{H} = -K \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

N_0 atoms per well.





At $T=0$ quantum fluctuations dominate

Mott insulator transition at $K \approx U$

Condensates at each lattice site ($N_0 \gg 1$); phase ϕ_i

Finite Temperature

Thermal fluctuations dominate

$$\Delta\phi_{Th} = \sqrt{T/J}$$
$$\Delta\phi_Q = (U/4J)^{1/4}$$

Define $J = N_0 K$

Then for $N_0 \gg 1$ and $J/N_0^2 \ll U$

Bose-Hubbard → $\boxed{\hat{H} = -J\sum_{\langle i,j\rangle}\cos(\phi_i-\phi_j) - \frac{U}{2}\sum_j \frac{\partial^2}{\partial\phi_j^2}}$

This is the 'quantum phase model' Trombettoni, et al , *NJP* **7** (2005) 57

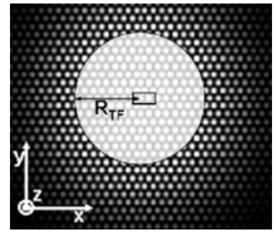
Quantum Monte Carlo shows

(i) At low temperatures, $kT \ll J$ all phases same

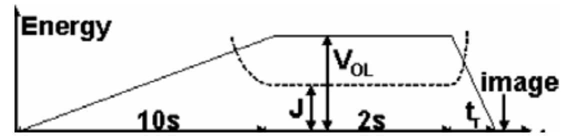
(ii) At $kT \approx J$ phases unlock. Identify as BKT transition

Recent experiment from Cornell's group

Schweikhard, Tung, Cornell, cond-mat 07040289

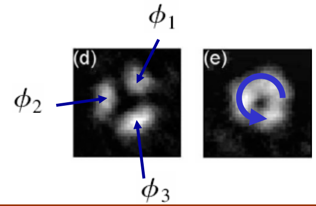


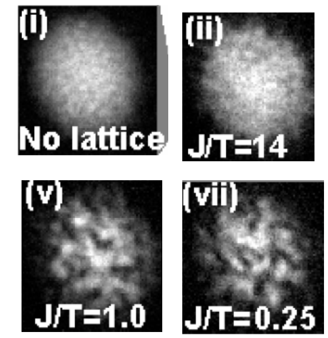
Experiment is in 2D hexagonal optical lattice



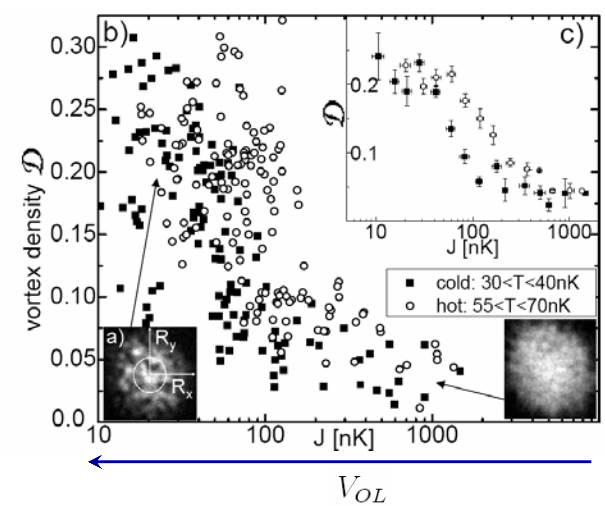
Closely related experiment from Anderson's group

PRL 98,110402,2007





Key result



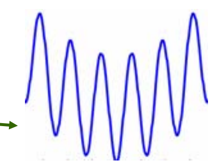
$N_{well} \approx 7000$

Our Classical field simulation

Dynamical evolution

Use projected GP in 2D

$$i\hbar\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \mathcal{P}\Big(-\frac{\hbar^2\nabla^2}{2m} + \underbrace{V_{\rm trap} + V_{\rm lattice}} + u|\psi|^2\Big)\psi(\mathbf{x},t)$$

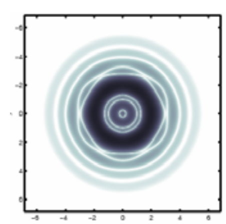


Initial state: Equilibrium state of harmonic trap at temperature T

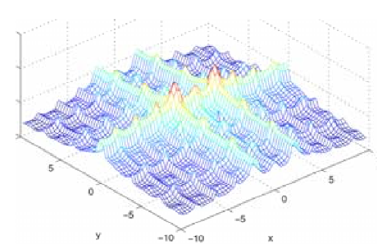
Technical issue: *What mode basis for evolution ?*

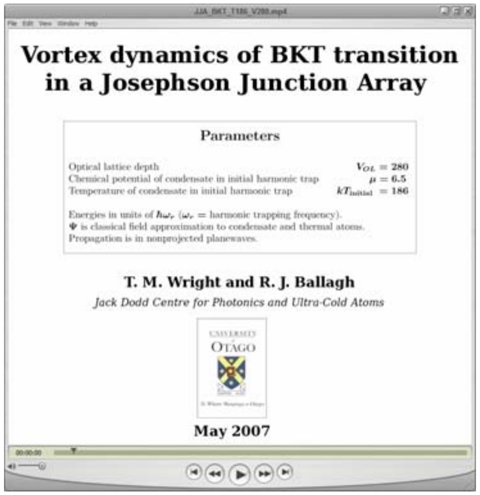
e.g. first excited state for $V_{\rm OL}$ *= 320*

Gauss-Laguerre basis



Plane wave basis





Parameters

V_{OL} = 280

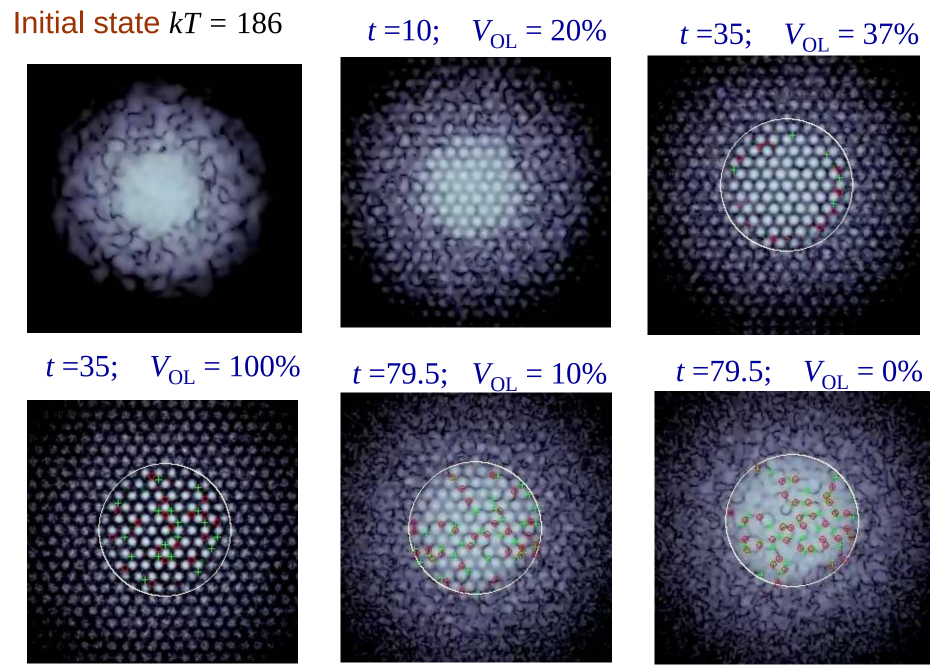
kT = 186

recoil energy = 3.8

initial μ = 6.5

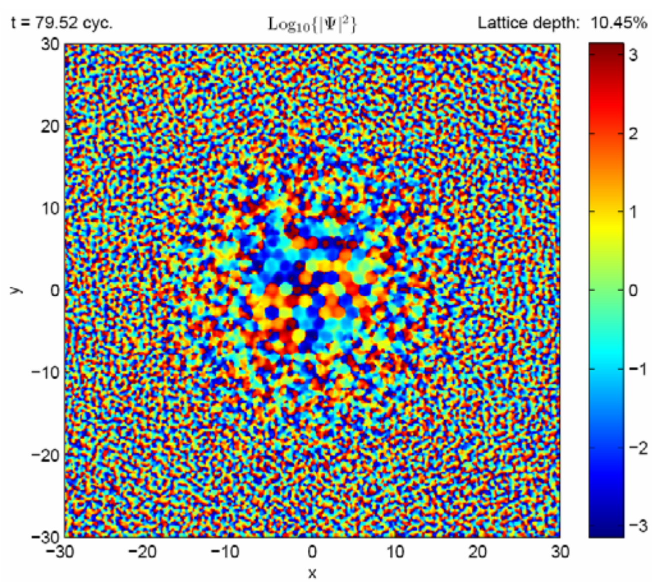
Units: in terms of Harmonic trap: [energy] $\hbar\omega$

Remark: some technical issues for constructing projector



Phase of Field

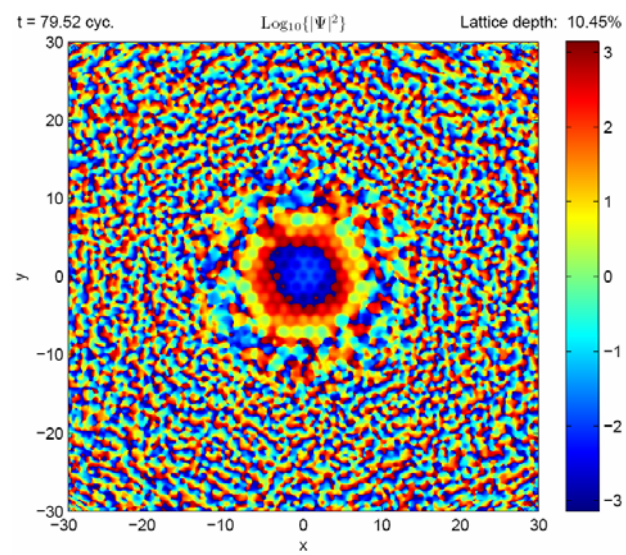
T=186 : Above the BKT transition



Phase of Field

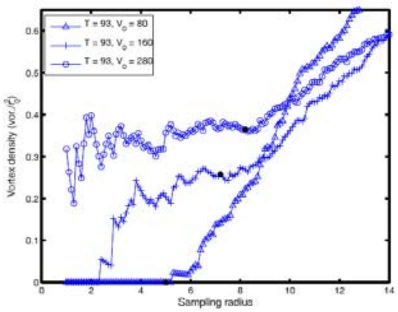
T=0 : Below the BKT transition

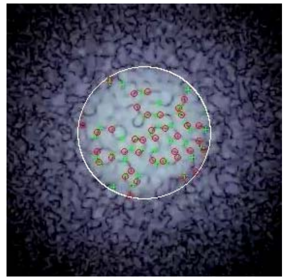
Quantum noise only



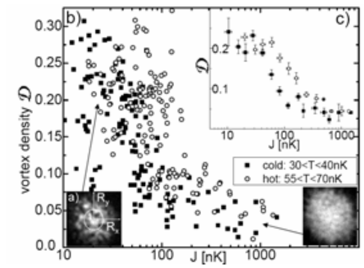
Vortex Counting

After lattice ramp-down





Experimental results



Summary

- Development and application of classical field methods for finite temperature condensates
- Revisit formation of vortex lattice by stirring
  - modelled development of thermal cloud
    - no vortex lattice formed
- BKT transition of condensates in a lattice
  - promising tool for quantitative description
  - projector needs to be developed